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One Electron Atom in Special Relativity with de Sitter Space-Time Symmetry

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Abstract

The de Sitter invariant Special Relativity (dS-SR) is a SR with constant curvature, and a natural extension of usual Einstein SR (E-SR). In this paper, we solved the dS-SR Dirac equation of Hydrogen by means of the adiabatic approach and the quasi-stationary perturbation calculations of QM. Hydrogen atoms are located on the light cone of the Universe. FRW metric and ΛCDM cosmological model are used to discuss this issue. To the atom, effects of de Sitter space-time geometry described by Beltrami metric are taken into account. The dS-SR Dirac equation turns out to be a time dependent quantum Hamiltonian system. We revealed that: 1,The fundamental physics constants m_e, $\hbar$, e variate adiabatically along with cosmologic time in dS-SR QM framework. But the fine-structure constant $\alpha \equiv e^2/(\hbar c)$ keeps to be invariant; 2,$(2s^{1/2} - 2p^{1/2})$-splitting due to dS-SR QM effects: By means of perturbation theory, that splitting $\Delta E(z)$ were calculated analytically, which belongs to $\mathcal{O}(1/R^2)$-physics of dS-SR QM. Numerically, we found that when $|R| \simeq \{10^3 Gly,\ 10^4 Gly,\ 10^5 Gly\ \}$, and $z \simeq \{1,\ \text{or } 2\}$, the $\Delta E(z) >> 1$(Lamb shift). This indicate that for these cases the hyperfine structure effects due to QED could be ignored, and the dS-SR fine structure effects are dominant. This effect could be used to determine the universal constant R in dS-SR, and be thought as a new physics beyond E-SR.

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1 Introduction

Einstein's Special Relativity (E-SR) has global Poincaré-Minkowski space-time symmetry. E-SR indicates the space-time metric is $\eta_{\mu\nu} = diag\{+,-,-,-\}$. The most general transformation to preserve metric $\eta_{\mu\nu}$ is Poincaré group. It is well known that the Poincaré group is the limit of the de Sitter group with the sphere radius $R \to \infty$. Thus people could pursue whether there exists another type of de Sitter transformation with $R \to finite$ which also leads to a Special Relativity theory (SR). In 1970's, Lu, Zou and Guo suggested the Special Relativity theory with de Sitter space-time symmetry (dS-SR) [1] [2]. In recent years, there are various studies of this theory [3]. In 2005, Yan, Xiao, Huang and Li performed Lagrangian-Hamiltonian formulism for dS-SR dynamics with two universal constants c and R, and suggested the quantum mechanics of dS-SR [4]. There is one universal parameter c (speed of light) in the Einstein's Special Relativity (E-SR). By contrast, there are two universal parameters in the de Sitter Special Relativity (dS-SR): c and R (the radius of de Sitter sphere and to character the cosmic radius). In this present paper, we try to study one-electron atoms, typically Hydrogen atom, of a distant galaxy (e.g., a Quasi-Stellar Object (QSO)) by means of dS-SR Quantum Mechanics (QM) suggested in Ref. [4].

As is well known that one of GR principles is existence of Locally Inertial System (LIS) at any point with small enough vicinity region in the curved space-time. In LIS, the expressions of physics laws are the same as ones in SR. Therefore determining the energy level shifts of a distant Hydrogen atom due to dS-SR QM will be useful to the cosmology when the curved space-time is the Friedmann-Robertson-Walker (FRW) Universe.

Ref. [4] shows that the dS-SR dynamical action for free particle associates the dynamics with time- and coordinates-dependent Hamiltonian. In other hand, the Noether theorem assures the symmetry's Neother charges to be conserved even though that the Hamiltonian is time- and space-dependent. In Ref. [4], 10 external conserved Neother charges for dS-SR have been explicitly presented, which are free particle's energy, 3 momenta, 3 angular-momenta and 3 boost generators (see, Eqs (52)-(56) in [4]). Thus, the energy conservation law in dS-SR holds, and at the same time the dS-SR dynamics is a time-dependent Hamiltonian system. Contrasting with E-SR dynamics, this is a remarkable feature of dS-SR. This will cause time-dependent level shifts in atomic physics, and lead to some remarkable observable effects in cosmology, for instance, the physics constants varying adiabatically [9] and some specific level shifts for Hydrogen atom caused by time interval bing on cosmic scale. In this paper, we will focus on the splitting effect between $2s^{1/2}$ and $2p^{1/2}$ states of Hydrogen in dS-SR Quantum Mechanics (QM).

In this paper, the adiabatic approach [5] [6] [7] [8] will be used to deal with the time-dependent Hamiltonian problems in dS-SR QM. Generally, to a $H(x,t)$, we may express it as $H(x,t) = H_0(x) + H'(x,t)$. Suppose two eigenstates $|s\rangle$ and $|m\rangle$ of $H_0(x)$ do not generate, i.e., $\Delta E \equiv \hbar(\omega_m - \omega_s) \equiv \hbar\omega_{ms} \neq 0$. The validness of for adiabatic approximation relies on the fact that the variation of the potential $H'(x,t)$ in the the Bohr time-period $(\Delta T_{ms}^{(Bohr)})\dot{H}'_{ms} = (2\pi/\omega_{ms})\dot{H}'_{ms}$ is much less than $\hbar\omega_{ms}$, where $H'_{ms} \equiv \langle m|H'(x,t)|s\rangle$. That makes the quantum transition from state $|s\rangle$ to state $|m\rangle$ almost impossible. Thus, the non-adiabatic effect corrections are small enough (or tiny) , and the adiabatic approximations are legitimate . To the wave equation of dS-SR QM of atoms discussed in this paper, we show that the perturbation Hamiltonian described the time evolutions of the system

$H'(x,t) \propto (c^2t^2/R^2)$ (where t is the cosmic time). Since R is cosmologically large and $R >> ct$, the factor (c^2t^2/R^2) will make the time-evolution of the system is so slow that the adiabatic approximation works. We shall provide a calculations to confirm this point in the paper. By means of this approach, we solve the stationary dS-SR Dirac equation for one electron atom, and the spectra of the corresponding Hamiltonian with time-parameter are obtained. Consequently, we find out that the electron mass m_e, the electric charge e and the Planck constant $\hbar$ vary as cosmic time going by, but the fine structure constant $\alpha = e^2/(\hbar c)$ keeps to be invariant. Those are interesting consequences since they indicate that the time-variations of fundamental physics constants are due to well known quantum evolutions of time-dependent quantum mechanics that has been widely discussed for a long history (e.g., see [7] and the references within).

The life time of a stable atom, e.g., the Hydrogen atom, is almost infinitely long. We can practically compare the spectra of atoms at nowadays laboratories to ones emitted (or absorbed) from the atoms of a distant galaxy, typically a Quasi-Stellar Object (QSO). The time interval could be on the cosmic scales. Such observation of spectra of distant astrophysical objects may encode some cosmologic information in the atomic energy levels at the position and time of emission. As is well known that the solutions of E-SR Dirac equation of atom are cosmologic effects free because the Hamiltonian of E-SR is time-independent, and the solutions at any time are of the same. Thus, after deducting Hubble red shifts, any deviation of cosmology atom spectrum observations from the results of E-SR Dirac equation could attribute to some new physics beyond E-SR. dS-SR is one of the most straightforward answers to such kind of deviations.

In E-SR Dirac equation of Hydrogen, $2s^{1/2}$- and $2p^{1/2}$-states are completely degenerate. The hyperfine effects of QED break this degeneracy, and turn out famous "Lamb Shift": i.e., $E_{2s^{1/2}} - E_{2p^{1/2}} \equiv 1\ Lamb\ Shift = 4.35152 \times 10^{-6} eV = 1057.9 MHz \times 2\pi\hbar$. In this paper, we solve the dS-SR Dirac equation for one-electron atom and reveal a dS-SR effect which also contribute a level shift to break the $2s^{1/2} - 2p^{1/2}$-degeneracy. This shift is proportional to Q^1Q^0/R^2, where Q^1 is the distance between the Earth and an observed galaxy (e.g., a QSO), Q^0 is the corresponding time interval, and R is the radius of de Sitter sphere. When Q^1Q^0/R^2 were large enough, the $2s^{1/2} - 2p^{1/2}$ splitting due to dS-SR effects would be much larger than QED's Lamb shift, the observation of this splitting in cosmological experiments could provide a criteria to check dS-SR.

The contents of the paper are organized as follows: In section II, we briefly recall the classical mechanics of de Sitter special relativity and the corresponding quantum mechanics. The dS-SR Dirac equation for spin-1/2 is presented; Section III is devoted to discuss Hydrogen atom described by dS-QM and embedden in light cone of Friedmann-Robertson-Walker (FRW) Universe; Section IV shows the solutions of usual E-SR Dirac equation for Hydrogen atom located at distant galaxy. Especially, the wave functions and energy values of the states $2s^{1/2}$ and $2p^{1/2}$ are presented explicitly. The Hydrogen atom energy level shifts due to gravity in FRW Universe are estimated; In Section V, we derive dS-SR Dirac equation for Hydrogen atom up to terms being proportional to $1/R^2$; Section VI: dS-SR Dirac equation for spectra of Hydrogen atom; Section VII: Adiabatic approximation solution to dS-SR Dirac spectra equation and time variation of physical constants; Section VIII: $2s^{1/2} - 2p^{1/2}$ splitting in the dS-SR Dirac equation of Hydrogen; Finally, in Section IX, we briefly discuss and summarize the results of this paper. In Appendix A, we derive the electric Coulomb Law in

QSO-Light-Cone Space; In Appendix B, we show the calculations of adiabatic approximative wave functions in dS-SR-Dirac equation of Hydrogen in detail. In Appendix C, we provide analytic calculations to the matrix elements of perturbation Hamiltonian in dS-SR, which yield $(2S^{1/2} - 2p^{1/2})$-hyperfine splitting discussed in the text.

2 Special Relativity with de Sitter Symmetry and dS-SR Dirac equation

The precise dS-SR theory were formulated in 1970–1974 by LU, ZOU and GUO [1] [2] (for the English version, see, e.g., Ref. [4] [3]). Two theorems were proved:

Lemma I: Inertial motion law for free particles holds to be true in the space-time characterized by Beltrami metric

$$B_{\mu\nu}(x) = \frac{\eta_{\mu\nu}}{\sigma(x)} + \frac{1}{R^2\sigma(x)^2}\eta_{\mu\lambda}\eta_{\nu\rho}x^\lambda x^\rho, \tag{1}$$

where $\sigma(x) \equiv 1 - \frac{1}{R^2}\eta_{\mu\nu}x^\mu x^\nu$, and the constant R is the radius of the pseudo-sphere in de Sitter (dS) space. $R^2 > 0$ or < 0 that corresponds to dS symmetries $SO(4,1)$ or $SO(3,2)$ respectively. This claim means that in the space-time characterized by $B_{\mu\nu}$, the velocity of free particle is constant, i.e.,

$$\dot{\mathbf{x}} = \overrightarrow{v} = constant, \qquad \text{for free particle} \tag{2}$$

which is exactly the counterpart of E-SR's inertial law in Minkowski space characterized by $\eta_{\mu\nu}$. (see Refs. [3] [4] for the English version of proof to Eq.(2)).

Lemma II: The de Sitter space-time transformation preserving $B_{\mu\nu}(x)$ is as follows

$$\begin{aligned}
x^\mu \longrightarrow \tilde{x}^\mu &= \pm\sigma(a)^{1/2}\sigma(a,x)^{-1}(x^\nu - a^\nu)D^\mu_\nu, \\
D^\mu_\nu &= L^\mu_\nu + R^{-2}\eta_{\nu\rho}a^\rho a^\lambda(\sigma(a) + \sigma^{1/2}(a))^{-1}L^\mu_\lambda, \\
L : &= (L^\mu_\nu) \in SO(1,3), \\
\sigma(x) = 1 &- \frac{1}{R^2}\eta_{\mu\nu}x^\mu x^\nu, \quad \sigma(a,x) = 1 - \frac{1}{R^2}\eta_{\mu\nu}a^\mu x^\nu.
\end{aligned} \tag{3}$$

where x^μ is the coordinate in an initial Beltrami frame, and $\tilde{x}^\mu$ is in another Beltrami frame whose origin is a^μ in the original one. There are 10 parameters in the transformations between them. Under the transformation (3), we have the equation preserving $B_{\mu\nu}$ as follows

$$B_{\mu\nu}(x) \longrightarrow \widetilde{B}_{\mu\nu}(\widetilde{x}) = \frac{\partial x^\lambda}{\partial \widetilde{x}^\mu}\frac{\partial x^\rho}{\partial \widetilde{x}^\nu}B_{\lambda\rho}(x) = B_{\mu\nu}(\widetilde{x}). \tag{4}$$

(see Appendix of Ref. [4] for the English version of proof to Eq.(4)). Eq.(4) will yield conservation laws for the energy, momenta, angular momenta and boost chargers of particles in dS-SR mechanics [4]. Here, we like to address that the space-time symmetry described by Eqs. (3) (4) is a global symmetry since both a^μ and L^μ_ν are constants instead of functions of space-time x^μ. This situation is same as E-SR's, where the Poincaré symmetry is global.

Based on the dS-SR space time theory described in above two lemmas, the dS-SR dynamics described by the Lagrangian as follows [4]

$$L = -m_0c\frac{ds}{dt} = -m_0c\frac{\sqrt{B_{\mu\nu}(x)dx^\mu dx^\nu}}{dt} = -m_0c\sqrt{B_{\mu\nu}(x)\dot{x}^\mu\dot{x}^\nu}, \tag{5}$$

where $\dot{x}^\mu = \frac{d}{dt}x^\mu$, $B_{\mu\nu}(x)$ is Beltrami metric (1). Then the canonic momenta and canonic energy (i.e., Hamiltonian) reads

$$\pi_i = \frac{\partial L}{\partial \dot{x}^i} = -m_0\sigma(x)\Gamma B_{i\mu}\dot{x}^\mu \tag{6}$$

$$H = \sum_{i=1}^{3}\frac{\partial L}{\partial \dot{x}^i}\dot{x}^i - L = m_0 c\sigma(x)\Gamma B_{0\mu}\dot{x}^\mu, \tag{7}$$

where

$$\Gamma^{-1} = \sigma(x)\frac{ds}{cdt} = \frac{1}{R}\sqrt{(R^2-\eta_{ij}x^ix^j)(1+\frac{\eta_{ij}\dot{x}^i\dot{x}^j}{c^2}) + 2t\eta_{ij}x^i\dot{x}^j - \eta_{ij}\dot{x}^i\dot{x}^jt^2 + \frac{(\eta_{ij}x^i\dot{x}^j)^2}{c^2}}. \tag{8}$$

From the equation of motion $\delta L = 0$, we have [4]

$$\dot{\Gamma}|_{\ddot{x}^i=0} = 0, \tag{9}$$

whose corresponding one in E-SR is

$$\dot{\gamma}|_{\ddot{x}^i=0} \equiv \frac{d}{dt}\left(\frac{1}{\sqrt{1-v^2/c^2}}\right)\Bigg|_{v=\text{constant}} = 0. \tag{10}$$

By means of the standard procedure to perform the canonic quantization, we obtained the dS-SR wave equation for spinless particle [4]:

$$\frac{1}{\sqrt{B}}\partial_\mu(B^{\mu\nu}\sqrt{B}\partial_\nu)\phi + \frac{m_0^2c^2}{\hbar^2}\phi = 0, \tag{11}$$

which is just the Klein-Gordon equation in curved space-time with Beltrami metric $B_{\mu\nu}$. The measurable conserved 4-momentum operator is [4]

$$p^\mu = i\hbar\left[\left(\eta^{\mu\nu} - \frac{x^\mu x^\nu}{R^2}\right)\partial_\nu + \frac{5x^\mu}{2R^2}\right]. \tag{12}$$

The corresponding Dirac equation which describes the particle with spin 1/2 [10] [12] reads

$$\left(ie_a^\mu\gamma^a D_\mu - \frac{m_0c}{\hbar}\right)\psi = 0, \quad \text{or} \quad \left(ie_{a\mu}\gamma^a D^\mu - \frac{m_0c}{\hbar}\right)\psi = 0, \tag{13}$$

where e_a^μ is the tetrad and D_μ is the covariant derivative with Lorentz spin connection ω_μ^{ab}. Their definitions and relations are follows (e.g., see [12])

$$\begin{aligned}
D_\mu &= \partial_\mu - \frac{i}{4}\omega_\mu^{ab}\sigma_{ab}, \quad D^\mu = B^{\mu\nu}D_\nu, \\
\{\gamma^a,\gamma^b\} &= 2\eta^{ab}, \quad \sigma_{ab} = \frac{i}{2}[\gamma_a,\gamma_b], \quad \frac{i}{2}[\sigma_{ab},\sigma_{cd}] = \eta_{ac}\sigma_{bd} - \eta_{ad}\sigma_{bc} + \eta_{bd}\sigma_{ac} - \eta_{bc}\sigma_{ad}, \\
e_\mu^a e_\nu^b \eta_{ab} &= B_{\mu\nu}, \quad e_\mu^a e_\nu^b B^{\mu\nu} = \eta^{ab}, \quad e_{a\,;\nu}^{\mu} = \partial_\nu e_a^\mu + \omega_a{}^b{}_\nu e_b^\mu + \Gamma_{\lambda\nu}^\mu e_a^\lambda = 0, \\
\omega_\mu^{ab} &= \frac{1}{2}(e^{a\rho}\partial_\mu e_\rho^b - e^{b\rho}\partial_\mu e_\rho^a) - \frac{1}{2}\Gamma_{\lambda\mu}^\rho(e^{a\lambda}e_\rho^b - e^{b\lambda}e_\rho^a), \\
\Gamma_{\lambda\mu}^\rho &= \frac{1}{2}B^{\rho\nu}(\partial_\lambda B_{\nu\mu} + \partial_\mu B_{\nu\lambda} - \partial_\nu B_{\lambda\mu}).
\end{aligned} \tag{14}$$

It is straightforward to check that the components ψ_α $(\alpha = 1, \cdots 4)$ of the spinor satisfy the Klein-Gordon equation (11).

3 Hydrogen atom embedded in light cone of Friedmann-Robertson-Walker Universe

The isotropic and homogeneous cosmology solution of Einstein equation in GR (General Relativity) is Friedmann-Robertson-Walker (FRW) metric. In this section we discuss the Hydrogen atom embedded in FRW Universe and described by dS-SR Dirac equation.

As is well known that GR can be viewed as a kind of gauge field theory [10] [11]. The dynamic equation of GR can be yielded by means of the localization of external global space-time symmetries, such as Lorentz group $SL(2,C)$, transition group T_4, Pioncaré group etc. Such localizations make global space -time transformation $x^\mu \to \tilde{x}'^\mu = \Lambda^\mu{}_\nu x^\nu + a^\mu$ to be $x^\mu \to \tilde{x}'^\mu = \Lambda^\mu{}_\nu(x) x^\nu + a^\mu(x) = f^\mu(x)$ which is arbitrary curvilinear coordinate transformation with Christoffel symbol connection $\{{}^{\lambda}_{\mu\nu}\}$ for the torsion-free space-time, and leads to construct GR by means of Riemann tensor. It is essential that the the framework of GR is generally free to the gauge theory's underline global external space time symmetry for the torsion-free space-time. Now let's see the global de Sitter space-time transformation Eq.(3). When $a^\mu \to a^\mu(x)$, $L^\mu_\nu \to L^\mu_\nu(x)$, the transformation is localizated to be $x^\mu \to \tilde{x}^\mu = f(x)^\mu$ which is an arbitrary curvilinear coordinate transformation. The connection is still Christoffel symbol and no corrections are yielded to the GR framework, and hence the considerations of atoms described by dS-SR QM in FRW Universe are legitimate.

One way to detect the spectrum of distance atom is spectroscopic observations of gas clouds seen in absorption against background Quasi-Stellar Objects (QSO), which can be used to search for level shifts of atom for various purposes (see, e.g., [13] [14]). In the observations of gas-QSO systems in the expanding Universe, one can observe two species of frequency changes in atomic spectra: the Hubble redshift (z) caused by the usual Doppler effects and a rest frequency change due to the dynamics of atom beyond E-SR. The latter can be found by measuring the relative size of relativistic corrections to the transition frequencies of atoms on the gas-QSO (or on QSO for briefness). A widely accepted assumption is that this rest frequency change is independent of the Hubble velocity and cosmologic acceleration of the gas in the Universe. Therefore all relativistic correction calculations for atom spectra were performed in a "rest" inertial reference frame without any Lorentz boost and non-inertial effects caused by frame-origin motion [15,16]. In this present paper, the calculations based on dS-SR Dirac equation are performed in such rest reference frame. In this framework, the time-varying $(2s^{1/2} - 2p^{1/2})$-splitting $\Delta E(2s^{1/2} - 2p^{1/2}) = \Delta E(t)$ will be calculated. Furthermore, the $t - z$ relation has been established for the Universe with the observed acceleration in ΛCDM model [18, 20]. Employing this relation, we have $\Delta E(t) = \Delta E(t(z))$.

Now, we show the dS-SR Dirac equation of Hydrogen atom on a QSO in the Earth-QSO reference frame. As illustrated in FIG.1, the Earth is located at the origin of frame, the proton (nucleus of Hydrogen atom) is located at $Q = \{Q^0 \equiv ct,\ Q^1,\ Q^2 = 0,\ Q^3 = 0\}$. To an observable atom in four-dimensional space-time, the proton has to be located at QSO-light-cone with cosmic metric $g_{\mu\nu}$. Namely, Q must satisfy following light-cone equation (see FIG. 1 and set $Q^2 = Q^3 = 0$ for simplification)

$$ds^2 = g_{\mu\nu}(Q) dQ^\mu dQ^\nu = 0, \tag{15}$$

which determines $Q^1 = f(Q^0)$. We emphasize that the underlying space-time symmetry for the atom near Q described by dS-SR dynamics is de Sitter group instead of to limit it as

Poincaré symmetry of E-SR as usual, which is only a special limit of dS-SR's. The corresponding space-time metric is $B_{\mu\nu}(Q) = \eta_{\mu\nu}\left(1 + \frac{(Q^0)^2-(Q^1)^2}{R^2}\right) + \frac{1}{R^2}\eta_{\mu\lambda}Q^\lambda\eta_{\nu\rho}Q^\rho + \mathcal{O}(1/R^4)$ (see Eq.(1)). Note $B_{\mu\nu}(Q)$ is position Q-dependent, and Lorentz metric $\eta_{\mu\nu}$ for E-SR is not. Explicitly, from Eq.(1), we have

$$B_{\mu\nu}(Q) = \begin{pmatrix} 1+\frac{2(Q^0)^2-(Q^1)^2}{R^2} & -\frac{Q^0Q^1}{R^2} & 0 & 0 \\ -\frac{Q^1Q^0}{R^2} & -1+\frac{2(Q^1)^2-(Q^0)^2}{R^2} & 0 & 0 \\ 0 & 0 & -1-\frac{(Q^0)^2-(Q^1)^2}{R^2} & 0 \\ 0 & 0 & 0 & -1-\frac{(Q^0)^2-(Q^1)^2}{R^2} \end{pmatrix} \quad (16)$$

$$B^{\mu\nu}(Q) = \begin{pmatrix} 1-\frac{2(Q^0)^2-(Q^1)^2}{R^2} & \frac{Q^0Q^1}{R^2} & 0 & 0 \\ \frac{Q^1Q^0}{R^2} & -1-\frac{2(Q^1)^2-(Q^0)^2}{R^2} & 0 & 0 \\ 0 & 0 & -1+\frac{(Q^0)^2-(Q^1)^2}{R^2} & 0 \\ 0 & 0 & 0 & -1+\frac{(Q^0)^2-(Q^1)^2}{R^2} \end{pmatrix} \quad (17)$$

In following, we will solve Eq.(15) to determine Q^1 in FRW Universe model.

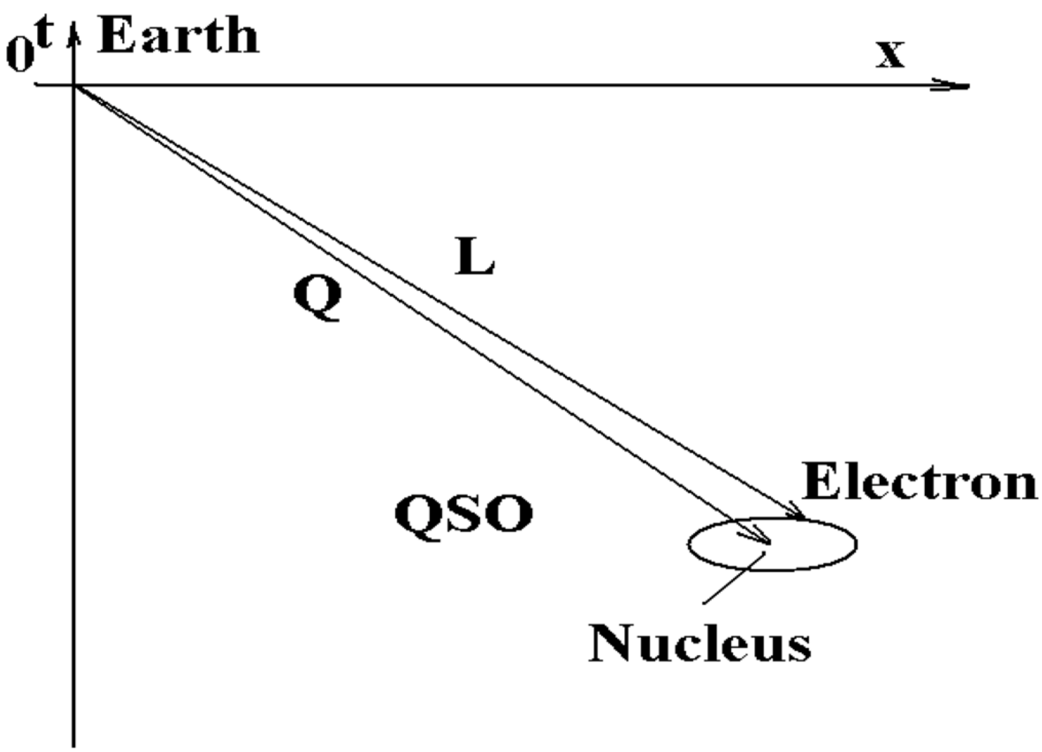

Figure 1: Sketch of the Earth-QSO reference frame. The Earth is located in the origin. The position vector for nucleus of atom on QSO is Q, and for electron is L. The distance between nucleus and electron is r.

The Friedmann-Robertson-Walker (FRW) metric is (see, e.g., [17])

$$\begin{aligned} ds^2 &= c^2dt^2 - a(t)^2\left\{\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\right\} \\ &= (dQ^0)^2 - a(t)^2\left\{dQ^idQ^i + \frac{k(Q^idQ^i)^2}{1-kQ^iQ^i}\right\} \\ &\equiv g_{\mu\nu}(Q)dQ^\mu dQ^\nu, \end{aligned} \quad (18)$$

where $r=\sqrt{Q^iQ^i}$, $Q^1=r\sin\theta\cos\phi$, $Q^2=r\sin\theta\sin\phi$, $Q^3=r\cos\theta$ has been used. As is well know FRW metric satisfies homogeneity and isotropy principle of present day cosmology. When $Q^2=Q^3=0$, from (18), we have

$$\begin{aligned} g_{\mu\nu}(Q) &= \eta_{\mu\nu}-a(t)^2\delta_{\mu1}\delta_{\nu1}\left(-\frac{1}{a(t)^2}+1+\frac{k(Q^1)^2}{1-k(Q^1)^2}\right) \\ &\quad -(a(t)^2-1)(\delta_{\mu2}\delta_{\nu2}+\delta_{\mu3}\delta_{\nu3}). \end{aligned} \tag{19}$$

For simpleness, we take $k=0$ and $a(t)=1/(1+z(t))$ (i.e., $a(t_0)=1$). And the red shift function $z(t)$ is determined by ΛCDM model [18–20](see, e.g., Eq.(64) of [19]):

$$t=\int_0^z\frac{dz'}{H(z')(1+z')}, \tag{20}$$

where

$$\begin{aligned} H(z') &= H_0\sqrt{\Omega_{m0}(1+z')^3+\Omega_{R0}(1+z')^4+1-\Omega_{m0}}, \\ H_0 &= 100\ h\simeq 100\times 0.705 km\cdot s^{-1}/Mpc, \\ \Omega_{m0} &\simeq 0.274, \quad \Omega_{R0}\sim 10^{-5}. \end{aligned} \tag{21}$$

Figure of $t(z)$ of Eq.(20) is shown in FIG.2.

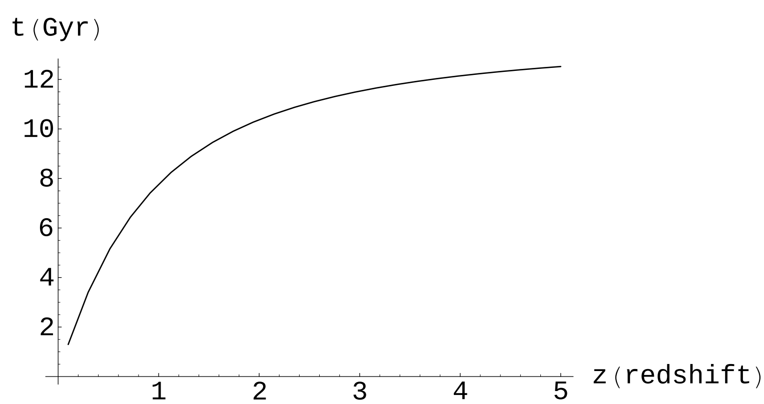

Figure 2: The $t-z$ relation in ΛCDM model (eq.(20)).

From (19), the FRW metric reads

$$g_{\mu\nu}(Q)=\eta_{\mu\nu}-(a(t)^2-1)(\delta_{\mu1}\delta_{\nu1}+\delta_{\mu2}\delta_{\nu2}+\delta_{\mu3}\delta_{\nu3}). \tag{22}$$

Substituting (22) into (15), we have

$$dQ^0=\sqrt{\frac{-g_{11}}{g_{00}}}dQ^1=a(t)dQ^1=\frac{1}{1+z(t)}dQ^1. \tag{23}$$

Consequently, by using Eq.(20) and $Q^0=c\,t$, we get desirous result:

$$Q^1=c\int_0^z\frac{dz'}{H(z')}. \tag{24}$$

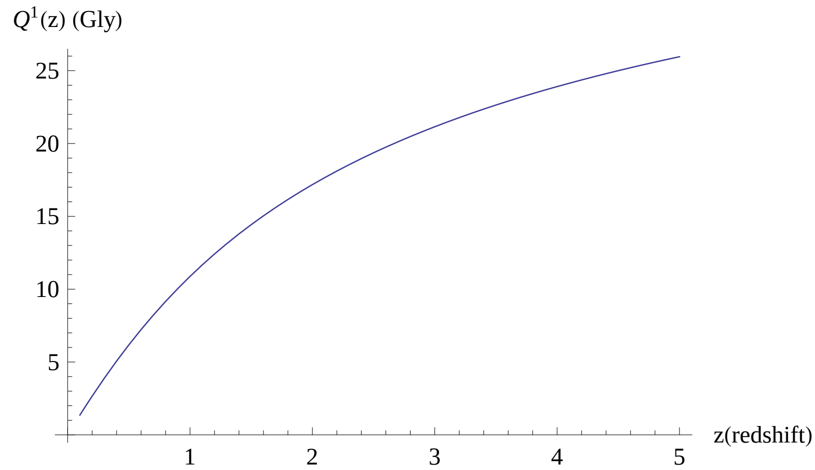

Figure 3: Function $Q^1(z)$ in ΛCDM model (eq.(24)).

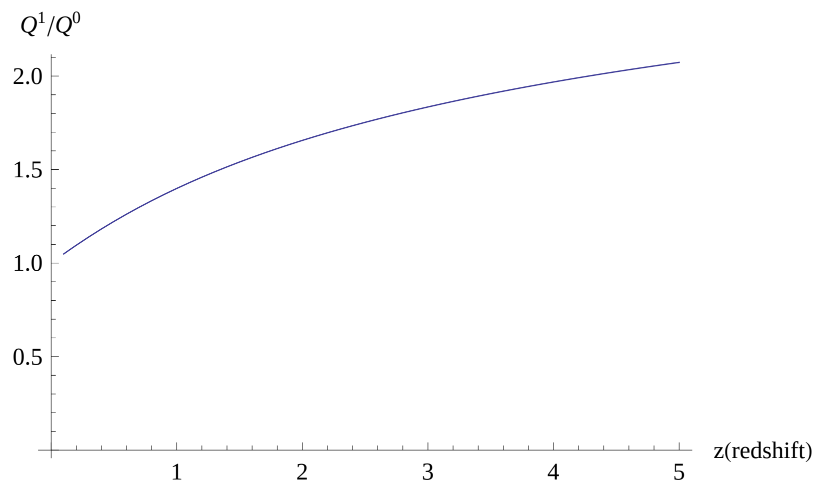

Figure 4: Function of $Q^1(z)/Q^0(z)$. $Q^1(z)$ and $Q^0(z) = ct$ are given in Eqs. (24) and (20).

Figure of $Q^1(z)$ of Eq.(24) is shown in FIG.3. Ratio of Q^1 over Q^0 is shown in FIG.4.

Then the location of distance proton is $\{Q^0, Q^1, 0, 0\}$ in the space-time with FRW metric.

We treat Hydrogen atom as a proton-electron bound state described by quantum mechanics under instantaneous approximations (see FIG. 1). The electron's coordinates are $L = \{L^0 \equiv ct_L \simeq ct,\ L^1,\ L^2,\ L^3\}$, and the relative space coordinates between proton and electron are $x^i = L^i - Q^i$. The magnitude of $r \equiv \sqrt{-\eta_{ij}x^i x^j} \sim a_B$ (where $a_B \simeq 0.5 \times 10^{-10}m$ is Bohr radius), and $|x^i| \sim a_B$.

According to gauge principle, the electrodynamic interaction between the nucleus and the electron can be taken into account by replacing the operator D^μ in eq.(13) with the $U(1)$-gauge covariant derivative $\mathcal{D}^\mu_L \equiv D^\mu_L - ie/(c\hbar)A^\mu$, where $A^\mu = \{\phi_B,\ \mathbf{A}\}$. Hence, the dS-SR Dirac equation for electron in Hydrogen at QSO reads

$$(ie_{\mu a}\gamma^a \mathcal{D}^\mu_L - \frac{\mu c}{\hbar})\psi = 0, \tag{25}$$

where $\mu = m_e/(1 + \frac{m_e}{m_p})$ is the reduced mass of electron, $\mathcal{D}^\mu_L = \frac{\partial}{\partial L_\mu} - \frac{i}{4}\omega^{ab\,\mu}\sigma_{ab} - ie/(c\hbar)A^\mu$, e^μ_a and ω^{ab}_μ have been given in eqs.(1) (14). For our purpose, we approximately write e^μ_a and

ω_μ^{ab} up to $\mathcal{O}(1/R^2)$ as follows:

$$e_a^\mu = \left(1 - \frac{\eta_{cd}L^c L^d}{2R^2}\right)\eta_a^\mu - \frac{\eta_{ab}L^b L^\mu}{2R^2} + \mathcal{O}(1/R^4), \tag{26}$$

$$\omega_\mu^{ab} = \frac{1}{2R^2}(\eta_\mu^a L^b - \eta_\mu^b L^a) + \mathcal{O}(1/R^4). \tag{27}$$

In the following sections we are going to solve dS-SR-Dirac equation for Hydrogen atom on QSO in FRW Universe model. In this quantum system, there are two cosmologic length scales: cosmic radius, say $R \sim 10^{12} ly$, and the distance between QSO and the Earth, that is about $\sim ct$: say $R > ct > 10^8 ly$, and two microcosmic length scales: the Compton wave length of electron $a_c = \hbar/(m_e c) \simeq 0.3 \times 10^{-12} m$, and Bohr radius $a_B = \hbar^2/(m_e e^2) \simeq 0.5 \times 10^{-10} m$. The calculations for our purpose will be accurate up to $\mathcal{O}(c^2t^2/R^2)$. The terms proportional to $\mathcal{O}(c^4t^4/R^4)$, $\mathcal{O}(cta_c/R^2)$, $\mathcal{O}(cta_B/R^2)$ etc will be omitted.

4 Solution of usual E-SR Dirac equation for hydrogen atom at QSO

4.1 Eigen values and eigen states

At first, we show the solution of usual E-SR Dirac equation in the Earth-QSO reference frame of Fig.1, which serves as leading order of solution for the dS-SR Dirac equation with $R \to \infty$ in that reference frame. For the Hydrogen, $\partial^\mu \to \mathcal{D}_L^\mu = \partial_L^\mu - ie/(c\hbar)A_M^\mu$ (noting $\omega^{ab\,\mu}|_{R\to\infty} = 0$), where $A_M^\mu \equiv \{\phi_M(x),\ \mathbf{A}_M(x)\}$, and $\phi_M(x)$ and $\mathbf{A}_M(x)$ are nucleus electric potential and vector potential at x^i in Minkowski space defined by following equations (see Appendix A, Eq.(139))

$$-\eta^{ij}\partial_i\partial_j\phi_M(x) = \nabla^2\phi_M(x) = -4\pi\rho(x) = -4\pi e\delta^{(3)}(\mathbf{x}), \tag{28}$$

$$\nabla(\partial_\lambda A_M^\lambda) - \partial^2\mathbf{A}_M = -\frac{4\pi}{c}\mathbf{j} = 0. \tag{29}$$

The solutions are $\phi_M(x) = e/r$ and $\mathbf{A}_M = 0$. And hence $\partial_0 \to \mathcal{D}_0^L = \partial_0 - i\eta_{00}e^2/(c\hbar r)$. Then, the E-SR Dirac equation reads

$$i\hbar\partial_t\psi = \left(-i\hbar c\vec{\alpha}\cdot\nabla_L + \mu c^2\beta - \frac{e^2}{r}\right)\psi, \tag{30}$$

where $\beta = \gamma^0,\ \alpha^i = \beta\gamma^i$. Noting the nucleus position $\mathbf{Q}$ =constant, we have

$$\nabla_L = \frac{\partial}{\partial\mathbf{L}} = \frac{\partial}{\partial(\mathbf{Q}+\mathbf{r})} = \frac{\partial}{\partial\mathbf{r}} \equiv \nabla, \tag{31}$$

and eq.(30) becomes the standard Dirac equation for electron in Hydrogen at its nucleus reference frame. Energy W for E-SR-mechanics is conserved (hereafter, we use notations of [22]), and the Hydrogen is the stationary states of E-SR Dirac equation. The stationary state condition is

$$i\hbar\partial_t\psi = W\psi. \tag{32}$$

As is well known, combining eqs.(30), (31) with (32), we have

$$W\psi = \left(-i\hbar c\vec{\alpha}\cdot\nabla + \mu c^2\beta - \frac{e^2}{r}\right)\psi \equiv H_0(r,\hbar,\mu,e)\psi, \tag{33}$$

which is the stationary E-SR Dirac equation for Hydrogen. The problem has been solved in terms of standard way, and the results are follows (see, e.g., [21] [22])

$$\begin{aligned} W = W_{n,\kappa} &= \mu c^2 \left(1+\frac{\alpha^2}{(n-|\kappa|+s)^2}\right)^{-1/2} \qquad (34)\\ &\alpha \equiv \frac{e^2}{\hbar c}, \qquad |\kappa| = (j+1/2) = 1,\ 2,\ 3\ \cdots \\ &s = \sqrt{\kappa^2-\alpha^2}, \qquad n = 1,\ 2,\ 3\ \cdots . \end{aligned}$$

And its expansion equation in α is

$$W = \mu c^2 \left(1 - \frac{\alpha^2}{2n^2} + \alpha^4\left(\frac{3}{8n^2} - \frac{1}{2n^3|\kappa|}\right) + \cdots\right). \qquad (35)$$

The corresponding Hydrogen's wave functions ψ have also already been finely derived (see e.g., [22]). The complete set of commutative observables is $\{H,\ \kappa,\ \mathbf{j}^2,\ j_z\}$, so that $\psi = \psi_{n,\kappa,j,j_z}(\mathbf{r},\hbar,\mu,\alpha) \equiv \psi_j^{m_j}(\mathbf{r})$, where $\mathbf{j} = \mathbf{L} + \frac{\hbar}{2}\tilde{\sigma}$, $\hbar\kappa = \beta(\tilde{\sigma}\cdot\mathbf{L}+\hbar)$, and $\alpha = e^2/(\hbar c)$. The expression of $\psi_j^{m_j}(\mathbf{r})$ is as follows

$$\psi_j^{m_j}(\mathbf{r}) = \begin{pmatrix} g_\kappa(r)\chi_\kappa^{m_j}(\hat{\mathbf{r}}) \\ i f_\kappa(r)\chi_{-\kappa}^{m_j}(\hat{\mathbf{r}}) \end{pmatrix} \qquad (36)$$

where

$$\begin{aligned} \chi_\kappa^{m_j}(\hat{\mathbf{r}}) &= \sum_{m_s=-1/2}^{1/2} C^{j\ m_j}_{l,m_j-m_s;\ 1/2,m_s} Y_l^{m_j-m_s}(\hat{\mathbf{r}})\chi^{m_s} \qquad (37)\\ \textit{with} \quad & \chi^{m_s=1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi^{m_s=-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ \chi_{-\kappa}^{m_j}(\hat{\mathbf{r}}) &= -\tilde{\sigma}_r\chi_\kappa^{m_j}(\hat{\mathbf{r}}), \quad \textit{with} \quad \tilde{\sigma}_r = \hat{\mathbf{r}}\cdot\overrightarrow{\sigma} = \begin{pmatrix} \cos\theta & \sin\theta e^{-i\phi} \\ \sin\theta e^{i\phi} & -\cos\theta \end{pmatrix}, \qquad (38) \end{aligned}$$

and

$$\begin{aligned} g_\kappa(r) &= 2\lambda(k_C+W_C)^{1/2}e^{-\lambda r}(2\lambda r)^{s-1}\alpha_0' \\ &\times\left(n'M(1-n',2s+1,2\lambda r) + \left(\kappa - \frac{\alpha k_C}{\lambda}\right)M(-n',2s+1,2\lambda r)\right), \qquad (39)\\ f_\kappa(r) &= 2\lambda(k_C-W_C)^{1/2}e^{-\lambda r}(2\lambda r)^{s-1}\alpha_0' \\ &\times\left(n'M(1-n',2s+1,2\lambda r) - \left(\kappa - \frac{\alpha k_C}{\lambda}\right)M(-n',2s+1,2\lambda r)\right), \qquad (40) \end{aligned}$$

where

$$\begin{aligned} & k_C = \mu c/\hbar,\ \ W_C = W/(c\hbar),\ \ \lambda = (k_C^2 - W_C^2)^{1/2},\ \ s = \sqrt{\kappa^2-\alpha^2},\ \ n' = n - |\kappa|, \\ & \kappa = -(j(j+1) - l(l+1) + 1/4), \qquad (41) \end{aligned}$$

where W has been given in Eq.(34), and $M(a,b,z)$ is the confluent hypergeometric function:

$$\begin{aligned} & M(a,b,z) = 1 + \frac{az}{b} + \frac{(a)_2 z^2}{2!(b)_2} + \cdots \frac{(a)_n z^n}{n!(b)_n} + \cdots \\ & (a)_0 = 1, \quad (a)_n = a(a+1)(a+2)\cdots(a+n-1), \end{aligned}$$

and α' is the normalization constant required by

$$\int_0^\infty r^2(g_\kappa^2(r)+f_\kappa^2(r))dr = 1.$$

To Hydrogen-like one electron atoms with $Z>1$, the energy level formula and the eigen wave functions expressions are all the same as Eqs.(34)-(41) except $\alpha \to \xi = Z\alpha$. In FIG.5 the levels of one electron atom with $Z=92$ are shown.

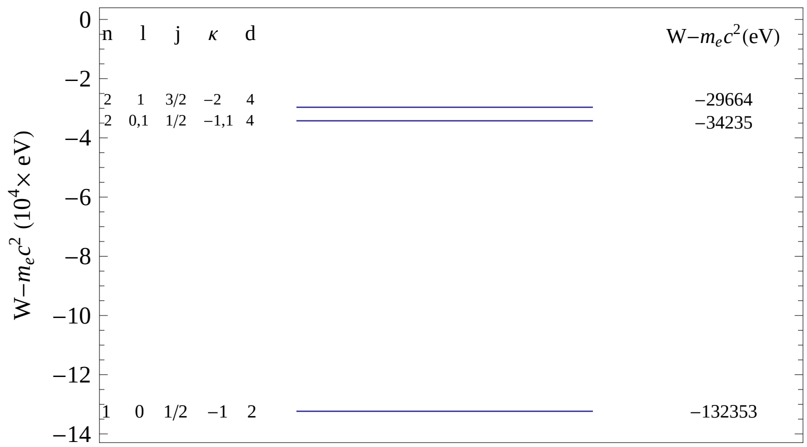

Figure 5: The energy levels of the one-electron atom with $Z=92$. The figure shows relativistic energy levels calculated using equation (34) with replacement of $\alpha \to \xi = Z\alpha$. They are labelled by the quantum numbers n, l, j, κ and their degeneracy d, up to $n=2$.

It is learned from above that since $R\to\infty$, $B_{\mu\nu}\to\eta_{\mu\nu}$, dS-SR→E-SR, the Hamiltonian of E-SR is cosmic time independent. So, the spectra of Hydrogen at any place in FRW Universe are the same, and there is almost no cosmology information of the Universe in the spectrum solutions Eq.(34) of E-SR Dirac equation.

4.2 $2s^{1/2}$ and $2p^{1/2}$ states of Hydrogen

As is well known that the state of $2s^{1/2}$ and state of $2p^{1/2}$ are complete degenerate to all order of α in the E-SR Dirac equation of Hydrogen. Namely, from (34) and $\kappa = -(j(j+1)-l(l+1)+1/4)$, we have

$$\Delta W(2s^{1/2}-2p^{1/2}) \equiv W_{(n=2,\,\kappa=-1)} - W_{(n=2,\,\kappa=+1)} = 0. \tag{42}$$

By means of Eqs.(36)-(40), the wave functions of states $2s^{1/2}$ with $\kappa=-1$ are as follows [22]

$$\psi^{m_j}_{(2s)j=1/2}(\mathbf{r}) = \begin{pmatrix} g_{(2s^{1/2})}(r)\chi^{m_j}_\kappa(\hat{\mathbf{r}})_{(2s^{1/2})} \\ if_{(2s^{1/2})}(r)\chi^{m_j}_{-\kappa}(\hat{\mathbf{r}})_{(2s^{1/2})} \end{pmatrix} \tag{43}$$

where

$$g_{(2s^{1/2})}(r) = \sqrt{\frac{(2\lambda)^{2s+1}k_C(2s+1)(k_C+W_C)}{2W_C(2W_C+k_C)\Gamma(2s+1)}}r^{s-1}e^{-\lambda r}\left(\frac{W_C}{k_C} - \frac{\lambda}{2s+1}\left(1+\frac{2W_C}{k_C}\right)r\right) \tag{44}$$

$$f_{(2s^{1/2})}(r) = -\sqrt{\frac{(2\lambda)^{2s+1}k_C(2s+1)(k_C-W_C)}{2W_C(2W_C+k_C)\Gamma(2s+1)}}r^{s-1}e^{-\lambda r}\left(\frac{W_C}{k_C}+1-\frac{\lambda}{2s+1}\left(1+\frac{2W_C}{k_C}\right)r\right) \tag{45}$$

where the expressions of notations $k_C,\ W_C,\ \lambda$ and s have been shown in Eq.(41) following [22], and due to Eqs.(37) (38)

$$\chi_{\kappa}^{1/2}(\hat{\mathbf{r}})_{(2s^{1/2})}=\begin{pmatrix} Y_0^0 \\ 0 \end{pmatrix},\ \chi_{\kappa}^{-1/2}(\hat{\mathbf{r}})_{(2s^{1/2})}=\begin{pmatrix} 0 \\ Y_0^0 \end{pmatrix},\ \ with\ \ Y_0^0=\frac{1}{\sqrt{4\pi}}, \tag{46}$$

$$\chi_{-\kappa}^{1/2}(\hat{\mathbf{r}})_{(2s^{1/2})}=\begin{pmatrix} -\cos\theta Y_0^0 \\ -\sin\theta e^{i\phi}Y_0^0 \end{pmatrix},\ \chi_{-\kappa}^{-1/2}(\hat{\mathbf{r}})_{(2s^{1/2})}=\begin{pmatrix} -\sin\theta e^{-i\phi}Y_0^0 \\ \cos\theta Y_0^0 \end{pmatrix}. \tag{47}$$

For the $2p^{1/2}$ state with $\kappa'=1$

$$\psi_{(2p)j=1/2}^{m_j}(\mathbf{r})=\begin{pmatrix} g_{(2p^{1/2})}(r)\chi_{\kappa'}^{m_j}(\hat{\mathbf{r}})_{(2p^{1/2})} \\ if_{(2p^{1/2})}(r)\chi_{-\kappa'}^{m_j}(\hat{\mathbf{r}})_{(2p^{1/2})} \end{pmatrix} \tag{48}$$

where

$$g_{(2p^{1/2})}(r)=\sqrt{\frac{(2\lambda)^{2s+1}k_C(2s+1)(k_C+W_C)}{2W_C(2W_C-k_C)\Gamma(2s+1)}}r^{s-1}e^{-\lambda r}\left(\frac{W_C}{k_C}-1+\frac{\lambda}{2s+1}\left(1-\frac{2W_C}{k_C}\right)r\right), \tag{49}$$

$$f_{(2p^{1/2})}(r)=-\sqrt{\frac{(2\lambda)^{2s+1}k_C(2s+1)(k_C-W_C)}{2W_C(2W_C-k_C)\Gamma(2s+1)}}r^{s-1}e^{-\lambda r}\left(\frac{W_C}{k_C}+\frac{\lambda}{2s+1}\left(1-\frac{2W_C}{k_C}\right)r\right) \tag{50}$$

$$\chi_{\kappa'}^{1/2}(\hat{\mathbf{r}})_{(2p^{1/2})}=\begin{pmatrix} -\sqrt{\frac{1}{3}}Y_1^0(\theta\phi) \\ \sqrt{\frac{2}{3}}Y_1^1(\theta\phi) \end{pmatrix},\ \chi_{\kappa'}^{-1/2}(\hat{\mathbf{r}})_{(2p^{1/2})}=\begin{pmatrix} -\sqrt{\frac{2}{3}}Y_1^{-1}(\theta\phi) \\ \sqrt{\frac{1}{3}}Y_1^0(\theta\phi) \end{pmatrix}, \tag{51}$$

$$\chi_{-\kappa'}^{1/2}(\hat{\mathbf{r}})_{(2p^{1/2})}=\begin{pmatrix} \sqrt{\frac{1}{3}}\cos\theta Y_1^0(\theta\phi)-\sqrt{\frac{2}{3}}\sin\theta e^{-i\phi}Y_1^1(\theta\phi) \\ \sqrt{\frac{1}{3}}\sin\theta e^{i\phi}Y_1^0(\theta\phi)+\sqrt{\frac{2}{3}}\cos\theta Y_1^1(\theta\phi) \end{pmatrix},$$

$$\chi_{-\kappa'}^{-1/2}(\hat{\mathbf{r}})_{(2p^{1/2})}=\begin{pmatrix} \sqrt{\frac{2}{3}}\cos\theta Y_1^{-1}(\theta\phi)-\sqrt{\frac{1}{3}}\sin\theta e^{-i\phi}Y_1^0(\theta\phi) \\ \sqrt{\frac{2}{3}}\sin\theta e^{i\phi}Y_1^{-1}(\theta\phi)+\sqrt{\frac{1}{3}}\cos\theta Y_1^0(\theta\phi) \end{pmatrix}, \tag{52}$$

where $Y_1^0(\theta\phi)=\sqrt{\frac{3}{4\pi}}\cos\theta,\ Y_1^1(\theta\phi)=-\sqrt{\frac{3}{8\pi}}e^{i\phi}\sin\theta,\ Y_1^{-1}(\theta\phi)=\sqrt{\frac{3}{8\pi}}e^{-i\phi}\sin\theta.$

4.3 Hydrogen atom energy level shifts due to gravity in FRW Universe

Secondly, we estimate the influence of external cosmological gravitational fields on the Hydrogen energy levels. 4D-FRW space-time is curved, and the atom locates in its tangent *flat* space-time described by $B_{\mu\nu}|_{R\to\infty}=\eta_{\mu\nu}$. It is well known that the interaction between an external gravitational field and atom may cause some energy level shifts of the atom [23] [24]. In [24], gravitational perturbations of the Hydrogen atom are derived by means of Fermi normal coordinate method [25]. It is found that energy level shift of one electron atom to the first order of curvature for nS states (see Eq.(8) in [24]):

$$E^{(1)}(nS)=\frac{1}{12}\zeta^{-2}\mu^{-1}n^2(5n^2+1)\mathcal{R}_{00}, \tag{53}$$

where $\mu^{-1}\zeta^{-1}=a_B$, $\zeta=e^2$ and $\mathcal{R}_{00}$ is Reimann curvature tensor. By means of FRW metric (22), Einstein equation (e.g., see Eq. (1.5.18) in *pp.36* of [17]) and ΛCDM, we have

$$\mathcal{R}_{00}=-3\ddot{a}/(c^2a)=\frac{4\pi}{c^2}G\rho_c(\Omega_{m0}-2\Omega_\Lambda)\simeq-1.05\times10^{-56}cm^{-2}, \tag{54}$$

and hence

$$
\begin{aligned}
E^{(1)}(nS) &= -\frac{1}{6}n^2(5n^2+1)\times 7.3\times 10^{-77}eV \\
&= -\frac{1}{6}n^2(5n^2+1)\times 1.7\times 10^{-71}(Lamb\ shift). \qquad (55)
\end{aligned}
$$

It is an extremely tiny level shift. To other energy levels, the corresponding level shifts are about same order magnitudes of Eq.(55). Physically, it is noting but a cosmologic gravity tide effects on the atom described by E-SR Dirac equation. Eq.(55) indicates that such tide effects can be completely ignored. Namely the Coulomb potential in the atom is determined by Eqs.(28) (29) which arise from Eq.(137) of Appendix A with $g_{\mu\nu} = \eta_{\mu\nu}$, and any influences of cosmological FRW metric are able to be ignored. Similar conclusion were reached in the de Sitter Universe model in [24]. In this present paper, though the atomic dynamics is of dS-SR Dirac equation rather than E-SR's, we will also ignore this sort of cosmological gravity tide effects to atoms. Namely, instead of $g_{\mu\nu} = \eta_{\mu\nu}$ for E-SR QM, we shall employe Beltromi metric $g_{\mu\nu} = B_{\mu\nu}$ to derive the Coulomb potential in the atom for dS-SR QM in the follows.

5 dS-SR Dirac equation for Hydrogen atom

By eqs.(25), (26), (27), $\partial^\mu \to \mathcal{D}^\mu_L = \frac{\partial}{\partial L_\mu} - \frac{i}{4}\omega^{ab\,\mu}\sigma_{ab} - ie/(c\hbar)A^\mu$, and noting $\mathcal{D}^L_\mu = B_{\mu\nu}\mathcal{D}^\nu_L$, we have the dS-SR Dirac equation for the electron in Hydrogen at the earth-QSO reference frame as follows

$$
\hbar c\beta\left[i\left(1-\frac{\eta_{ab}L^aL^b}{2R^2}\right)\gamma^\mu\mathcal{D}^L_\mu - \frac{i}{2R^2}\eta_{ab}L^a\gamma^bL^\mu\mathcal{D}^L_\mu - \frac{\mu c}{\hbar}\right]\psi = 0, \qquad (56)
$$

where factor $\hbar c\beta$ in the front of the equation is only for convenience. We expand each terms of (56) in order as follows:

1. Since observed QSO must be located on the light cone, then $\eta_{ab}L^aL^b \simeq \eta_{ab}Q^aQ^b = (Q^0)^2 - (Q^1)^2$, and the first term of (56) reads

$$
\hbar c\beta i\left(1-\frac{\eta_{ab}L^aL^b}{2R^2}\right)\gamma^\mu\mathcal{D}^L_\mu = \left(1-\frac{(Q^0)^2-(Q^1)^2}{2R^2}\right)\hbar c\beta i\gamma^\mu\mathcal{D}^L_\mu\psi \qquad (57)
$$

$$
\text{with}\ \ \beta\gamma^\mu = \{\beta\gamma^0 = \beta^2 = 1,\ \beta\gamma^i = \alpha^i\} \qquad (58)
$$

$$
\hbar c\beta i\gamma^\mu\mathcal{D}^L_\mu\psi = (i\hbar\partial_t + \hbar c\vec{\alpha}\cdot\nabla + \frac{\hbar c\beta}{4}\omega^{ab}_\mu\gamma^\mu\sigma_{ab} + e\beta\gamma^\mu B_{\mu\nu}A^\nu)\psi, \qquad (59)
$$

where $A^\nu = \{A^0 = \phi_B,\ A^i\}$ are determined by Maxwell equations within constance metric $g_{\mu\nu} = B_{\mu\nu}(Q)$ and $j^\nu = \{j^0 \equiv c\rho/\sqrt{B_{00}},\ j^i = 0\}$ (see Appendix A):

$$
\begin{aligned}
-B^{ij}(Q)\partial_i\partial_j\phi_B(x) &= \left[\left(1-\frac{(Q^0)^2-(Q^1)^2}{R^2}\right)\nabla^2 + \frac{(Q^1)^2}{R^2}\frac{\partial^2}{\partial(x^1)^2}\right]\phi_B(x) = -\frac{4\pi}{c}j^0 \\
&= \frac{-4\pi e}{\sqrt{-\det(B_{ij}(Q))B_{00}(Q)}}\delta^{(3)}(\mathbf{x}), \qquad (60)
\end{aligned}
$$

$$
\partial^i\partial_\mu A^\mu - B^{\mu\nu}\partial_\mu\partial_\nu A^i = -\frac{4\pi}{c}j^i = 0. \qquad (61)
$$

The solution is (see Appendix A)

$$\phi_B = \left(1 - \frac{3(Q^0)^2 - 2(Q^1)^2}{2R^2}\right)\frac{e}{r_B}, \quad A^i = 0, \tag{62}$$

where $r_B = \sqrt{(\tilde{x}^1)^2 + (x^2)^2 + (x^3)^2}$ with $\tilde{x}^1 \equiv x^1/[1 + \frac{(Q^1)^2}{2R^2}]$. In the follows, we use variables $\{\tilde{x}^1, x^2, x^3\}$ to replace $\{x^1, x^2, x^3\}$. Following notations are introduced hereafter:

$$\mathbf{r}_B = \mathbf{i}\tilde{x}^1 + \mathbf{j}x^2 + \mathbf{k}x^3, \quad |\mathbf{r}_B| = r_B, \tag{63}$$

$$\nabla_B = \mathbf{i}\frac{\partial}{\partial\tilde{x}^1} + \mathbf{j}\frac{\partial}{\partial x^2} + \mathbf{k}\frac{\partial}{\partial x^3}, \quad \tilde{x}^i \in \{\tilde{x}^1,\ x^2,\ x^3\}. \tag{64}$$

Then, noting $\frac{\partial}{\partial x^1} = \frac{\partial\tilde{x}^1}{\partial x^1}\frac{\partial}{\partial\tilde{x}^1} = \frac{\partial}{\partial\tilde{x}^1} - \frac{(Q^1)^2}{2R^2}\frac{\partial}{\partial\tilde{x}^1}$, the eq.(59) becomes

$$\begin{aligned}
\hbar c\beta i\gamma^\mu \mathcal{D}^L_\mu\psi &= \Big(i\hbar\partial_t + i\hbar c\vec{\alpha}\cdot\nabla_B - i\hbar c\frac{(Q^1)^2}{2R^2}\alpha^1\frac{\partial}{\partial\tilde{x}^1} \\
&\quad + \frac{\hbar c\beta}{4}\omega_\mu^{ab}\gamma^\mu\sigma_{ab} + eB_{00}\phi_B + e\alpha^1 B_{10}\phi_B\Big)\psi \\
&= \Big(i\hbar\partial_t + i\hbar c\vec{\alpha}\cdot\nabla_B - i\hbar c\frac{(Q^1)^2}{2R^2}\alpha^1\frac{\partial}{\partial\tilde{x}^1} + \frac{\hbar c\beta}{4}\omega_\mu^{ab}\gamma^\mu\sigma_{ab} \\
&\quad + \left[1 + \frac{2(Q^0)^2 - (Q^1)^2}{R^2}\right]e\phi_B - \frac{Q^1Q^0}{R^2}\alpha^1 e\phi_B\Big)\psi.
\end{aligned} \tag{65}$$

2. Estimation of the contributions of the fourth term in RSH of (65) (the spin-connection contributions) is as follows: By (27), the ratio of the fourth term to the first term of (65) is:

$$\left|\frac{\frac{\hbar c\beta}{4}\omega_\mu^{ab}\gamma^\mu\sigma_{ab}\psi}{i\hbar\partial_t\psi}\right| \sim \frac{\hbar c}{4}\frac{ct}{2R^2}\frac{1}{m_ec^2} = \frac{ct}{8R^2}\frac{\hbar}{m_ec} = \frac{1}{8}\frac{cta_c}{R^2} \sim 0, \tag{66}$$

where $a_c = \hbar/(m_ec) \simeq 0.3\times10^{-12}m$ is the Compton wave length of electron. $\mathcal{O}(cta_c/R^2)$-term is neglectable. Therefore the 3-rd term in RSH of (59) has no contribution to our approximation calculations.

3. Substituting (66) (65) (58) into (57) and noting $L^a \simeq Q^a$ (see FIG. 1), we get the first term in LHS of (56)

$$\begin{aligned}
\hbar c\beta i\left(1 - \frac{\eta_{ab}L^aL^b}{2R^2}\right)\gamma^\mu\mathcal{D}^L_\mu\psi &= \left(1 - \frac{(Q^0)^2 - (Q^1)^2}{2R^2}\right)\Big(i\hbar\frac{\partial}{\partial t} + i\hbar c\vec{\alpha}\cdot\nabla_B \\
&\quad - i\hbar c\frac{(Q^1)^2}{2R^2}\alpha^1\frac{\partial}{\partial\tilde{x}^1} + [1 + \frac{2(Q^0)^2 - (Q^1)^2}{R^2}]e\phi_B - \frac{Q^1Q^0}{R^2}\alpha^1 e\phi_B\Big)\psi \\
&= \Big\{\left(1 - \frac{(Q^0)^2 - (Q^1)^2}{2R^2}\right)\left(i\hbar\frac{\partial}{\partial t} + i\hbar c\vec{\alpha}\cdot\nabla_B\right) - i\hbar c\frac{(Q^1)^2}{2R^2}\alpha^1\frac{\partial}{\partial\tilde{x}^1} \\
&\quad + \left[1 + \frac{3(Q^0)^2 - (Q^1)^2}{2R^2}\right]e\phi_B - \frac{Q^1Q^0}{R^2}\alpha^1 e\phi_B + \mathcal{O}(1/R^4)\Big\}\psi.
\end{aligned} \tag{67}$$

4. The second term of (56) is

$$
\begin{aligned}
-\hbar c\beta\frac{i}{2R^2}\eta_{ab}L^a\gamma^b L^\mu \mathcal{D}^L_\mu\psi &= -\frac{i\hbar c\beta}{2R^2}(\gamma^0 L^0-\vec{\gamma}\cdot\vec{L})L^\mu\left[\partial^L_\mu-\delta_{\mu 0}\frac{ie}{c\hbar}\phi_B\right]\psi+\mathcal{O}(\frac{1}{R^4})\\
&= -\frac{i\hbar c}{2R^2}(L^0-\vec{L}\cdot\vec{\alpha})\left(L^0\partial^L_0-L^0\frac{ie}{c\hbar}\phi_B+L^i\partial^L_i\right)\psi\\
&\simeq -\frac{ic\hbar}{2R^2}(L^0-L^1\alpha^1)(L^0\partial^L_0-L^0\frac{ie}{c\hbar}\phi_B+L^1\partial^L_1)\psi\\
&\simeq -\frac{ic\hbar}{2R^2}\Big[\left((L^0)^2-L^1L^0\alpha^1\right)\partial_0-\frac{ie}{c\hbar}(L^0)^2\phi_B\\
&\quad +\frac{ie}{c\hbar}L^0L^1\alpha^1\phi_B+L^0L^1\partial^L_1-(L^1)^2\alpha^1\partial^L_1\Big]\psi, \qquad (68)
\end{aligned}
$$

where following approximation estimations were used

$$
\frac{(L^2)}{R}\sim\frac{(L^3)}{R}\sim\frac{a_B}{R}\sim 0. \qquad (69)
$$

Noting $L^0\simeq Q^0,\ L^1\simeq Q^1$, (68) becomes

$$
\begin{aligned}
-\hbar c\beta\frac{i}{2R^2}\eta_{ab}L^a\gamma^b L^\mu \mathcal{D}^L_\mu\psi &= \Big[\left(\frac{-(Q^0)^2}{2R^2}+\frac{Q^1Q^0}{2R^2}\alpha^1\right)i\hbar\partial_t-\frac{(Q^0)^2}{2R^2}e\phi_B+\frac{Q^1Q^0}{2R^2}\alpha^1 e\phi_B\\
&\quad -\frac{Q^1Q^0}{2R^2}i\hbar c\frac{\partial}{\partial\tilde{x}^1}+\frac{(Q^1)^2}{2R^2}i\hbar c\alpha^1\frac{\partial}{\partial\tilde{x}^1}\Big]\psi. \qquad (70)
\end{aligned}
$$

5. Therefore, substituting (67) (70) into (56), we have

$$
\begin{aligned}
i\hbar\left(1-\frac{2(Q^0)^2-(Q^1)^2}{2R^2}+\frac{Q^1Q^0}{2R^2}\alpha^1\right)\partial_t\psi &= \Big[-i\hbar c\left(1-\frac{(Q^0)^2-(Q^1)^2}{2R^2}\right)\vec{\alpha}\cdot\nabla_B+\mu c^2\beta\\
&\quad -\left(1+\frac{2(Q^0)^2-(Q^1)^2}{2R^2}\right)e\phi_B+\frac{Q^1Q^0}{2R^2}i\hbar c\frac{\partial}{\partial\tilde{x}^1}\Big]\psi, \qquad (71)
\end{aligned}
$$

or

$$
\begin{aligned}
i\hbar\partial_t\psi &= \Big[-\left(1+\frac{(Q^0)^2}{2R^2}\right)i\hbar c\vec{\alpha}\cdot\nabla_B+\left(1+\frac{2(Q^0)^2-(Q^1)^2}{2R^2}\right)\mu c^2\beta\\
&\quad -\left(1+\frac{(Q^0)^2}{2R^2}\right)\frac{e^2}{r_B}\Big]\psi-\frac{Q^1Q^0}{2R^2}\alpha^1\left[-i\hbar c\vec{\alpha}\cdot\nabla_B+\mu c^2\beta-\frac{e^2}{r_B}\right]\psi\\
&\quad +\left[\frac{Q^1Q^0}{2R^2}i\hbar c\frac{\partial}{\partial\tilde{x}^1}\right]\psi, \qquad (72)
\end{aligned}
$$

where $\phi_B=(1-\frac{3(Q^0)^2-2(Q^1)^2}{2R^2})e/r_B$ were used (see Eq.(62)). Eq.(72) is dS-SR Dirac wave equation to the first order of $\mathcal{O}(\frac{c^2t^2}{R^2})$. Two remarks on (72) are as follows:

i) When $R\to\infty$, Eq.(72) goes back to usual E-SR Dirac equation of Hydrogen, which has been discussed in the last section.

ii) Eq.(72) is a time-dependent wave equation. It is somewhat difficult to deal with the time-dependent problems in quantum mechanics. Generally, there are two approximative approaches to discuss two extreme cases respectively: (i) The modification in states obtained by the wave equation depends critically on the time T during which the

modification of the system's "Hamiltonian" take place. For this case, one would use the sudden approach; And, (ii), for case that of a very slow modification of Hamiltonian, the adiabatic approach works [6]. To wave equation of (72), like the discussions in Introduction of this paper, since $|R|$ is cosmologically large and $|R| >> ct$, factor $\{(Q^0)^2/R^2,\ (Q^1)^2/R^2\} \propto (c^2t^2/R^2)$ makes the time-evolution of the system is so slow that the adiabatic approximation [5] may legitimately works. In the below (the subsection **E**), we will provide a calculations to confirm this point.

6 dS-SR spectra equation of hydrogen

In order to discuss the spectra of Hydrogen by dS-SR Dirac equation, we need to find out its solutions with certain physics energy E. By eq.(12), and being similar to (32), the dS-SR-energy eigen-state condition for (72) can be derived by means of the operator expression of momentum in dS-SR (12):

$$\begin{aligned}
p^0 = \frac{E}{c} &= i\hbar\left[\frac{1}{c}\partial_t - \frac{ct}{R^2}x^\nu\partial^L_\nu + \frac{5ct}{2R^2}\right] \\
E &= i\hbar\left[\partial_t - \frac{c^2t^2}{R^2}\partial_t + \frac{5ct}{2R^2}\right] \\
E\psi &\simeq i\hbar\left(1-\frac{c^2t^2}{R^2}\right)\partial_t\psi = i\hbar\left(1-\frac{(Q^0)^2}{R^2}\right)\partial_t\psi,
\end{aligned} \tag{73}$$

where a estimation for the ratio of the 3-rd term to the 2-nd of $E\psi$ were used:

$$\frac{|i\hbar\frac{5c^2t}{2R^2}\psi|}{|\frac{-c^2t^2}{R^2}i\hbar\partial_t\psi|} \sim \frac{|i\hbar\frac{5c^2t}{2R^2}|}{|\frac{-2c^2t^2}{R^2}E|} \sim \frac{5\hbar}{2tm_ec^2} \equiv \frac{5}{2}\frac{a_c}{ct}$$

where $a_c \simeq 0.3\times 10^{-12}$m is the Compton wave length of electron and ct is about the distance between earth and QSO. In our approximative calculations $a_c/(ct)$ is neglectable. For instance, to a QSO with $ct \sim 10^9$ly, $a_c/(ct) \sim 10^{-38} << (ct)^2/R^2 \sim 10^{-5}$. Hence the 3-rd term of $E\psi$ were ignored.

Inserting (73) into (72), we obtain the $\mathcal{SR}_{cR}$-spectra equation of hydrogen

$$\begin{aligned}
\left(1+\frac{(Q^0)^2}{R^2}\right)E\psi &= \Bigg[-\left(1+\frac{(Q^0)^2}{2R^2}\right)i\hbar c\vec{\alpha}\cdot\nabla_B + \left(1+\frac{2(Q^0)^2-(Q^1)^2}{2R^2}\right)\mu c^2\beta \\
&\quad -\left(1+\frac{(Q^0)^2}{2R^2}\right)\frac{e^2}{r_B}\Bigg]\psi - \frac{Q^1Q^0}{2R^2}\alpha^1\left[-i\hbar c\overrightarrow{\alpha}\cdot\nabla_B + \mu c^2\beta - \frac{e^2}{r_B}\right]\psi \\
&\quad + \left[\frac{Q^1Q^0}{2R^2}i\hbar c\frac{\partial}{\partial\tilde{x}^1}\right]\psi,
\end{aligned}$$

or

$$
\begin{aligned}
E\psi &= \left[-\left(1-\frac{(Q^0)^2}{2R^2}\right) i\hbar c\vec{\alpha}\cdot\nabla_B + \left(1-\frac{(Q^1)^2}{2R^2}\right)\mu c^2\beta \right.\\
&\quad \left. -\left(1-\frac{(Q^0)^2}{2R^2}\right)\frac{e^2}{r_B}\right]\psi - \frac{Q^1Q^0}{2R^2}\alpha^1\left[-i\hbar c\overrightarrow{\alpha}\cdot\nabla_B+\mu c^2\beta-\frac{e^2}{r_B}\right]\psi\\
&\quad +\left[\frac{Q^1Q^0}{2R^2}i\hbar c\frac{\partial}{\partial\tilde{x}^1}\right]\psi\\
&= \left[-i\hbar_t c\vec{\alpha}\cdot\nabla_B+\mu_t c^2\beta-\frac{e_t^2}{r_B}\right]\psi-\frac{Q^1Q^0}{2R^2}\alpha^1\left[-i\hbar c\overrightarrow{\alpha}\cdot\nabla_B+\mu c^2\beta-\frac{e^2}{r_B}\right]\psi\\
&\quad +\left[\frac{Q^1Q^0}{2R^2}i\hbar c\frac{\partial}{\partial\tilde{x}^1}\right]\psi\\
&\equiv (\ H_0(r_B,\hbar_t,\mu_t,e_t)+H'\ )\psi\equiv H_{(dS-SR)}\psi,
\end{aligned}
\tag{74}
$$

which is up to $\mathcal{O}(c^2t^2/R^2)$ (say again, $\mathcal{O}(1/R^4)$, $\mathcal{O}(cta_B/R^2)$, $\mathcal{O}(cta_c/R^2)$ terms have been ignored), and where

$$
H_0(r_B,\hbar_t,\mu_t,e_t) = -i\hbar_t c\vec{\alpha}\cdot\nabla_B+\mu_t c^2\beta-\frac{e_t^2}{r_B}, \tag{75}
$$

$$
H' = \frac{Q^1Q^0}{2R^2}\left(-\alpha^1 H_0(r_B,\hbar_t,\mu_t,e_t)+i\hbar c\frac{\partial}{\partial\tilde{x}^1}\right) \tag{76}
$$

with

$$
\hbar_t = \left(1-\frac{(Q^0)^2}{2R^2}\right)\hbar = \left(1-\frac{c^2t^2}{2R^2}\right)\hbar, \tag{77}
$$

$$
\mu_t = \left(1-\frac{(Q^1)^2}{2R^2}\right)\mu, \tag{78}
$$

$$
e_t = \left(1-\frac{(Q^0)^2}{4R^2}\right)e = \left(1-\frac{c^2t^2}{4R^2}\right)e, \tag{79}
$$

and

$$
\alpha_t \equiv \frac{e_t^2}{\hbar_t c} = \frac{e^2}{\hbar c} = \alpha. \tag{80}
$$

Using notation in [22], $E_0 = W$ and the unperturbed eigen-state equation is

$$
W\psi = H_0(\hbar_t,\mu_t,e_t)\psi = \left(-i\hbar_t c\vec{\alpha}\cdot\nabla_B+\mu_t c^2\beta-\frac{e_t^2}{r_B}\right)\psi, \tag{81}
$$

which is same as (33) except $\hbar,\ \mu,\ e$ being replaced by $\hbar_t,\ \mu_t,\ e_t$. However, since the time t is dynamic variable in the time-dependent Hamiltonian system, at this present stage we do not know whether t can be approximately treated as a parameter in the system. Hence, we cannot yet conclude $\hbar,\ \mu,\ e$ are time variations described by (77), (78), (79) at this stage. In the following section, we pursue this subject.

7 Adiabatic approximation solution to dS-SR-Dirac spectra equation and time variation of physical constants

Comparing (81) with (33), we can see that there are three correction terms in (81), which are proportional to (c^2t^2/R^2) and service of effects of dS-SR QM. Since $R >> ct$, we argue

that the corrections due to the effects should be small, and the adiabatic approach works quite well for solving this QM problem. In order to being sure on this point, we examine the corrections beyond adiabatic approximations in below by calculating them explicitly for a certain z. Suppose $z \simeq 3 \sim 4$, FIG.(4) indicates $Q^1 \approx 1.7Q^0 = 1.7\ ct$, and hence $(Q^1)^2 \approx 3(Q^0)^2 = 3\ c^2t^2$ in (81). Rewriting this spectra equation (81) in version of wave equation like eq.(30) via $E \Rightarrow i\hbar\partial_t$, we have

$$\begin{aligned} i\hbar\partial_t\psi &= H(t)\psi = [H_0(r_B, \hbar, \mu, e) + H_0'(t)]\psi, && (82) \\ \text{where} \qquad H_0(r_B, \hbar, \mu, e) &= -i\hbar c\vec{\alpha}\cdot\nabla_B + \mu c^2\beta - \frac{e^2}{r_B} \quad (see\ eq.(33)) && (83) \\ H_0'(t) &= -\left(\frac{c^2t^2}{2R^2}\right) H_0(r_B, \hbar, 3\mu, e). && (84) \end{aligned}$$

Suppose initial state of the atom is $\psi(t=0) = \psi_s(\mathbf{r}_B, \hbar, \mu, \alpha)$ where $s = \{n_s,\ \kappa_s,\ \mathbf{j}_s^2,\ j_{sz}\}$, by eqs. (82) (83) (84), and catching the time-evolution effects, we have (see Chapter XVII of Vol II of [6], and Appendix B)

$$\psi(t) \simeq \psi_s(\mathbf{r}_B, \hbar_t, \mu_t, e_t)e^{-i\frac{W_s}{\hbar}t} + \sum_{m\neq s} \frac{\dot{H}_0'(t)_{ms}}{i\hbar\omega_{ms}^2}\left(e^{i\omega_{ms}t} - 1\right)\psi_m(\mathbf{r}_B, \hbar_t, \mu_t, e_t)e^{\left(-i\int_0^t \frac{W_m(\theta)}{\hbar}d\theta\right)}, \quad (85)$$

where $\psi_s(\mathbf{r}_B, \hbar_t, \mu_t, e_t)$ is the adiabatic wave function, $\hbar_t$, μ_t e_t are given in (77)– (79), and

$$\begin{aligned} \dot{H}_0'(t)_{ms}|_{(m\neq s)} &= \langle m|\dot{H}_0'(t)|s\rangle|_{(m\neq s)} = \frac{-c^2t}{R^2}\langle m|H_0(r_B, \hbar, 3\mu, e)|s\rangle|_{(m\neq s)} \\ &= \frac{-c^2t}{R^2}\langle m|\left(H_0(r_B, \hbar, 3\mu, e) - H_0(r_B, \hbar, \mu, e)\right)|s\rangle|_{(m\neq s)} \\ &\simeq \frac{-c^2t}{R^2}\langle n_m,\ \kappa_m,\ \mathbf{j}_m^2,\ j_{mz}|2\mu c^2\beta|n_s,\ \kappa_s,\ \mathbf{j}_s^2,\ j_{sz}\rangle e^{-i(\omega_s-\omega_m)t} \\ &= \frac{-2\mu c^4t}{R^2}\langle m|\beta|s\rangle e^{-i(\omega_s-\omega_m)t}, && (86) \\ \omega_{ms} &= \omega_m - \omega_s, \quad \omega_m = \frac{W_m}{\hbar}. && (87) \end{aligned}$$

Note, formula $\langle m|H_0(r,e)|s\rangle|_{m\neq s} = 0$ has been used in the calculations of (86). The second term of Right-Hand-Side (RHS) of eq. (85) represents the quantum transition amplitudes from ψ_s-state to ψ_m, which belong to the correction effects beyond adiabatic approximations. Now for showing the order of magnitude of such corrections, we estimate $|\dot{H}_0'(t)_{ms}/\hbar\omega_{ms}^2|$ for $s = 1 \equiv (1s^{1/2}, \kappa = -1, m_j = 1/2)$, $m = 2 \equiv (2s^{1/2}, \kappa = -1, m_j = 1/2)$. Noting $W_n \approx \mu c^2 - \mu c^2\alpha^2/(2n^2)$, and the Compton wave length of electron $a_c = \hbar/(m_ec) \simeq \hbar/(\mu c) \simeq 0.3\times 10^{-12}m$, from Eqs.(86)(87) we have

$$\left|\frac{\dot{H}_0'(t)_{21}}{\hbar\omega_{21}^2}\right| = \left|\frac{128}{9\alpha^4}\langle 2|\beta|1\rangle\right|\frac{a_c}{R}\frac{ct}{R}, \quad with\ \ \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad (88)$$

where the state $\langle 2|$ has been given in Eqs.(43)-(50) and the state $|1\rangle$ is as follows: [22]

$$|1\rangle = \psi_{(1s)j=1/2}^{m_j=1/2}(\mathbf{r}) = \begin{pmatrix} g_{(1s^{1/2})}(r)\chi_\kappa^{1/2}(\hat{\mathbf{r}})_{(1s^{1/2})} \\ if_{(1s^{1/2})}(r)\chi_{-\kappa}^{1/2}(\hat{\mathbf{r}})_{(1s^{1/2})} \end{pmatrix} \quad (89)$$

where

$$g_{(1s^{1/2})}(r) = \sqrt{\frac{(2\lambda_1)^{2s+1}(k_C + W_{C1})}{2k_C\Gamma(2s+1)}}\, r^{s-1}e^{-\lambda_1 r} \tag{90}$$

$$f_{(1s^{1/2})}(r) = -\sqrt{\frac{(2\lambda_1)^{2s+1}(k_C - W_{C1})}{2k_C\Gamma(2s+1)}}\, r^{s-1}e^{-\lambda_1 r} \tag{91}$$

where $W_{C1} \simeq m_e c^2(1-\alpha^2/2)/(c\hbar)$, $\lambda_1 = \sqrt{k_C^2 - W_{C1}^2}$ and

$$\chi_\kappa^{1/2}(\hat{\mathbf{r}})_{(1s^{1/2})} = \begin{pmatrix} Y_0^0 \\ 0 \end{pmatrix}, \quad \chi_{-\kappa}^{1/2}(\hat{\mathbf{r}})_{(1s^{1/2})} = \begin{pmatrix} -\cos\theta Y_0^0 \\ -\sin\theta e^{i\phi} Y_0^0 \end{pmatrix}. \tag{92}$$

By means of expressions of $\psi_{(1s)j=1/2}^{m_j=1/2}(\mathbf{r})$ (Eqs.(89)-(92)) and $\psi_{(2s)j=1/2}^{m_j=1/2}(\mathbf{r})$ (Eqs.(43)-(50)), we have

$$\langle 2|\beta|1\rangle = \int_0^\infty dr r^2 \left(g_{(1s^{1/2})}(r)g_{(2s^{1/2})}(r) - f_{(1s^{1/2})}(r)f_{(2s^{1/2})}(r)\right) \simeq -1.12\times 10^{-5}. \tag{93}$$

Substituting (93) into (88), we obtain

$$\left|\frac{\dot{H}_0'(t)_{21}}{\hbar\omega_{21}^2}\right| = 5.6\times 10^4 \times \frac{a_c}{R}\frac{ct}{R} \tag{94}$$

Considering Compton wave length of electron $a_c = \frac{\hbar}{m_e c} \simeq 0.3\times 10^{-12} m \simeq 0.3\times 10^{-28} ly$ and both $Q^0 = ct$ and R are cosmological large length scales, and $R > Q^0$, therefore we have that

$$\left|\frac{\dot{H}_0'(t)_{21}}{\hbar\omega_{21}^2}\right| \simeq \frac{(1.7\times 10^{-24} ly)}{R} \times \frac{Q^0}{R} << 1. \tag{95}$$

To generic $\langle m|$ and $|s\rangle$, similar to the calculations of Eq.(94), we always have

$$\left|\frac{\dot{H}_0'(t)_{ms}}{\hbar\omega_{ms}^2}\right| \simeq (constant)\cdot\frac{a_c}{R}\times\frac{Q^0}{R}. \tag{96}$$

Since the $(constant)$ at last is about $\sim 10^{10}$, and then $\left|\frac{\dot{H}_0'(t)_{ms}}{\hbar\omega_{ms}^2}\right| \simeq \frac{(1.7\times 10^{-18} ly)}{R}\times\frac{Q^0}{R}$, we finally obtain

$$\left|\frac{\dot{H}_0'(t)_{ms}}{\hbar\omega_{ms}^2}\right| << 1, \tag{97}$$

which indicates the second term of adiabatic expansion expression Eq.(85) can be ignored, or the corrections from beyond adiabatic approximations are quite small (or tiny), and hence the adiabatic approximation is legitimate for solving this dS-SR Dirac equation of the atom.

Thus, we achieve an interesting consequence that the fundamental physics constants variate adiabatically along with cosmologic time in dS-SR quantum mechanics framework. As is well known that the quantum evolution in the time-dependent quantum mechanics has been widely accepted and studied during past several decades (see, e.g., [7]). It is remarkable that the time-variations of μ (or m_e), $\hbar$ and e (see Eqs. (77) (78) (79)) belong to such quantum evolution effects.

8 $2s^{1/2}$-$2p^{1/2}$ splitting in the dS-SR Dirac equation of Hydrogen

In the Section IV, we have pointed that the state of $2s^{1/2}$ and state of $2p^{1/2}$ are complete degenerate to all order of α in the E-SR Dirac equation of Hydrogen described in Hamiltonian $H_0(r,\hbar,\mu,e)$ of Eq.(33). In this section, we calculate the $(2S^{1/2}-2p^{1/2})$-splitting duo to dS-SR effects.

8.1 Energy levels shifts of Hydrogen in dS-SR QM as perturbation effects of E-SR Dirac equation of atom

The dS-SR Dirac spectrum equation for Hydrogen atom has been derived in Section VI (74), which is as follows

$$H_{(dS-SR)}\psi = (\ H_0(r,\hbar_t,\mu_t,e_t) + H'\ )\psi = E\psi, \tag{98}$$

where

$$H_0(r,\hbar_t,\mu_t,e_t) = -i\hbar_t c\vec{\alpha}\cdot\nabla + \mu_t c^2\beta - \frac{e_t^2}{r}, \tag{99}$$

$$H' = \frac{1}{2}(H'^{\dagger} + H') \equiv H'_1 + H'_2, \tag{100}$$

where

$$H'_1 = -\frac{Q^1Q^0}{4R^2}\left(\alpha^1 H_0(r,\hbar_t,\mu_t,e_t) + H_0(r,\hbar_t,\mu_t,e_t)\alpha^1\right), \tag{101}$$

$$H'_2 = \frac{Q^1Q^0}{4R^2}\left(i\hbar c\overrightarrow{\frac{\partial}{\partial x^1}} - i\hbar c\overleftarrow{\frac{\partial}{\partial x^1}}\right). \tag{102}$$

Comparing above equations with Eqs. (74)$-$(76), hereafter we have removed the subscript B of r_B and ∇_B, and the tilde notation $\sim$ of $\tilde{x}^1$ for simplicity. And we have also rewritten the perturbation Hamiltonian H' to be explicit hermitian. In the spherical polar coordinates system the operator $\frac{\partial}{\partial x^1} \equiv \partial_1$ in Eq.(101) is as follows

$$\partial_1 = \frac{\partial}{\partial x^1} = \vec{i}\cdot\nabla = \sin\theta\cos\phi\frac{\partial}{\partial r} + \cos\theta\cos\phi\frac{1}{r}\frac{\partial}{\partial\theta} - \frac{\sin\phi}{r\sin\theta}\frac{\partial}{\partial\phi}. \tag{103}$$

Comparing dS-SR QM with ordinary E-SR Dirac equation of Hydrogen, there are two distinguishing effects in the dS-SR QM descriptions of distant Hydrogen atom (or one-electron atom) in the Earth-QSO reference frame: 1, The physical constants variations with cosmic time adiabatically, which have been discussed in the previous section (see Eqs. (77)$-$(79)); 2, Perturbation effects arisen from H' of Eq.(100) in $H_{(dS-SR)}$ of Eq.(98). In this section, we focuss the latter.

For adiabatic quantum system, the states are quasi-stationary in all instants. And hence to all instants the quasi-stationary perturbation theory works. When $H_{(dS-SR)} = H_0(r,\hbar_t,\mu_t,e_t) + H'$, the unperturbed quasi-stationary solutions of $H_0(r,\hbar_t,\mu_t,e_t)\psi = W\psi$ are the same as Eqs.(33)$-$(41) except $\hbar \to \hbar_t,\ \mu \to \mu_t,\ e \to e_t$. Then the energy levels shifts due to H' of Eq.(100) are computable in practice by the perturbation approach in QM.

Those shifts $\Delta E^i \equiv W_i - E$ due to H' are determined by

$$\det\left(\langle H_{(dS-SR)}\rangle_{ii'} - E\delta_{ii'}\right) = 0, \tag{104}$$

where $i = \{n,\ l,\ j,\ \kappa,\ m_j\}$, $H_{(dS-SR)}$ has been given in Eq.(98) and the elements are

$$\langle H_{(dS-SR)}\rangle_{ii'} = \langle i|H_0|i'\rangle + \langle i|H'|i'\rangle = W_i\delta_{ii'} + \langle H'\rangle_{ii'}, \tag{105}$$

where $W_i = W_{n,\kappa}$ are shown in Eq.(34).

Firstly, we compute the elements $\langle H'\rangle_{ii'} \equiv \langle (H_1' + H_2')\rangle_{ii'}$ with $i = i'$. From Eq.(101), we have

$$\begin{aligned}
\langle H_1'\rangle_{ii} &= -\frac{Q^1Q^0}{2R^2}W_i\langle i|\alpha^1|i\rangle \\
&= -\frac{Q^1Q^0}{2R^2}W_i\int drr^2\int d\Omega\left(g_\kappa(r)\chi_\kappa^{m_j\dagger}(\hat{\mathbf{r}}), -if_\kappa(r)\chi_{-\kappa}^{m_j\dagger}(\hat{\mathbf{r}})\right)\begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0\end{pmatrix}\begin{pmatrix} g_\kappa(r)\chi_\kappa^{m_j}(\hat{\mathbf{r}}) \\ if_\kappa(r)\chi_{-\kappa}^{m_j}(\hat{\mathbf{r}})\end{pmatrix} \\
&\propto \int d\Omega\left(\chi_\kappa^{m_j\dagger}(\hat{\mathbf{r}})\sigma^1\chi_{-\kappa}^{m_j}(\hat{\mathbf{r}}) - \chi_{-\kappa}^{m_j\dagger}(\hat{\mathbf{r}})\sigma^1\chi_\kappa^{m_j}(\hat{\mathbf{r}})\right).
\end{aligned} \tag{106}$$

Substituting Eqs.(37) (38) into Eq.(106), we get

$$\int d\Omega\chi_\kappa^{m_j\dagger}(\hat{\mathbf{r}})\sigma^1\chi_{-\kappa}^{m_j}(\hat{\mathbf{r}}) = 0 \tag{107}$$

$$\int d\Omega\chi_{-\kappa}^{m_j\dagger}(\hat{\mathbf{r}})\sigma^1\chi_\kappa^{m_j}(\hat{\mathbf{r}}) = 0, \tag{108}$$

and hence

$$\langle H_1'\rangle_{ii} = 0. \tag{109}$$

It is easy to be sure the validness of Eqs.(107) (108). Considering, for instance, the case of state $|i\rangle = |(2p^{1/2})^{m_j=1/2}\rangle$ (see Eqs. (51) (52)), we can calculate the left sides of (107) (108) explicitly:

$$\begin{aligned}
&\int d\Omega\chi_{\kappa=1}^{1/2\dagger}(\hat{\mathbf{r}})_{(2p^{1/2})}\sigma^1\chi_{-\kappa=-1}^{1/2}(\hat{\mathbf{r}})_{(2p^{1/2})} \\
&= \int d\Omega\left(-\sqrt{\frac{1}{3}}Y_1^0(\theta\phi)^\dagger, \sqrt{\frac{2}{3}}Y_1^1(\theta\phi)^\dagger\right)\begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix}\begin{pmatrix} \sqrt{\frac{2}{3}}\cos\theta Y_1^{-1}(\theta\phi) - \sqrt{\frac{1}{3}}\sin\theta e^{-i\phi}Y_1^0(\theta\phi) \\ \sqrt{\frac{2}{3}}\sin\theta e^{i\phi}Y_1^{-1}(\theta\phi) + \sqrt{\frac{1}{3}}\cos\theta Y_1^0(\theta\phi)\end{pmatrix} \\
&= \frac{-i}{2\pi}\int_0^{2\pi}d\phi\sin\phi\int_0^\pi d\theta\sin\theta\cos^2\theta(\sin\theta - \cos\theta) = 0,
\end{aligned} \tag{110}$$

$$\int d\Omega\chi_{-\kappa=-1}^{1/2\dagger}(\hat{\mathbf{r}})_{(2p^{1/2})}\sigma^1\chi_{\kappa=1}^{1/2}(\hat{\mathbf{r}})_{(2p^{1/2})} = \left(\int d\Omega\chi_{\kappa=1}^{1/2\dagger}(\hat{\mathbf{r}})_{(2p^{1/2})}\sigma^1\chi_{-\kappa=-1}^{1/2}(\hat{\mathbf{r}})_{(2p^{1/2})}\right)^\dagger = 0. \tag{111}$$

Therefore Eqs.(107) (108) hold to be true for the state of $|i\rangle = |(2p^{1/2})^{m_j=1/2}\rangle$.

From Eq.(102), we vave

$$\langle H_2'\rangle_{ii} = \frac{Q^1Q^0}{4R^2}\langle i|\left(i\hbar c\overrightarrow{\frac{\partial}{\partial x^1}} - i\hbar c\overleftarrow{\frac{\partial}{\partial x^1}}\right)|i\rangle = -\frac{Q^1Q^0}{2R^2}c\langle i|\hat{p}^1|i\rangle = 0, \tag{112}$$

where $\hat{p}^1 = -i\frac{\partial}{\partial x^1}$, and the fact that the average value for p^1 to the stationary bound state $|i\rangle$ must be vanish have been used. Eq.(112) can also be checked by explicit calculations based the known wave functions Eqs.(36)$-$(38).

Combining Eq.(112) with Eq.(110), we find out that

$$\langle H'\rangle_{ii} = \langle H_1'\rangle_{ii} + \langle H_2'\rangle_{ii} = 0, \tag{113}$$

which means $\langle H'\rangle_{ii'}$ is an off-diagonal matrix in the Hilbert space. As is well known that the energy shifts for non-degenerate levels due to H' are as

$$\Delta E^i \equiv W_i - E = \langle H'\rangle_{ii} + \sum_{i'\neq i} \frac{'|\langle H'\rangle_{i'i}|^2}{W_i - W_{i'}} + \cdots, \tag{114}$$

which could be thought as the perturbation solution of Eq.(104) for non-degenerate case. Thus, since $H' \propto \mathcal{O}(1/R^2)$ and noting Eq.(113), the non-degenerate level shifts ΔE^i are $\mathcal{O}(1/R^4)$, which are beyond the considerations of this paper. We will show in next subsection that the meaningful $\mathcal{O}(1/R^2)$-energy level shifts due to off-diagonal perturbation interaction H' are occurred for degeneration levels. Typical example is $2S^{1/2} - 2p^{1/2}$ splitting due to H'.

8.2 $\mathbf{2s^{1/2} - 2p^{1/2}}$ splitting caused by $\mathbf{H'}$

In the Section IV, we have shown that the state of $2s^{1/2}$ and state of $2p^{1/2}$ are complete degenerate to all order of α in the E-SR Dirac equation of Hydrogen (see Eq.(42)). The degenerate will be broken by the effects of H'. In this subsection we calculate the $2s^{1/2} - 2p^{1/2}$ splitting caused by H'.

By using the explicit expressions of $2s^{1/2}$- and $2p^{1/2}$ wave functions Eqs.(43)$-$(52), all matrix elements of $H' = H_1' + H_2'$ between them can be calculated, i.e,

$$\langle H'\rangle^{m_j,\, m_j'}_{2L^{1/2},2L'^{1/2}} = \langle H_1'\rangle^{m_j,\, m_j'}_{2L^{1/2},2L'^{1/2}} + \langle H_2'\rangle^{m_j,\, m_j'}_{2L^{1/2},2L'^{1/2}}, \tag{115}$$

where

$$\begin{aligned}\langle H_i'\rangle^{m_j,\, m_j'}_{2L^{1/2},2L'^{1/2}} &= \langle (2L^{1/2})^{m_j}|H_i'|(2L'^{1/2})^{m_j'}\rangle \\ &= \int drr^2 \int d\Omega\; \psi^{m_j\dagger}_{(2L)j=1/2}(\mathbf{r})\; H_i'\; \psi^{m_j'}_{(2L')j=1/2}(\mathbf{r}),\end{aligned} \tag{116}$$

where $\{L, L'\} = \{s, p\}$, $i = 1, 2$, and $\psi^{m_j}_{(2L)j=1/2}(\mathbf{r})$ are given in Eqs.(43)$-$(52). The matrix element calculations are presented in step by step in Appendix C, and the results are listed as follows:

1. H_1'-matrix elements: H_1' is given in (101), i.e.,

$$H_1' = -\frac{Q^1 Q^0}{4R^2}\left(\alpha^1 H_0(r, \hbar, \mu, e) + H_0(r, \hbar, \mu, e)\alpha^1\right), \tag{117}$$

where the subscript t of $\hbar, \mu, e$ has been removed, because there is already a factor of $(1/R^2)$ in the H_1'-expression and $1/R^4$-terms are ignorable. By means of straightforward calculations (see Appendix C), we have

$$\begin{aligned}&\langle H_1'\rangle^{1/2,\,-1/2}_{2s^{1/2},2p^{1/2}} = \langle H_1'\rangle^{-1/2,\,1/2}_{2s^{1/2},2p^{1/2}} = -\langle H_1'\rangle^{1/2,\,-1/2}_{2p^{1/2},2s^{1/2}} = -\langle H_1'\rangle^{-1/2,\,1/2}_{2p^{1/2},2s^{1/2}} \\ &= -i\frac{Q^1Q^0}{4R^2}\frac{W}{3}\sqrt{\frac{k_C^2 - W_C^2}{4W_C^2 - k_C^2}}\left(\frac{W_C}{k_C} - \frac{k_C}{2W_C}(s+1)\right) \equiv -i\Theta_1,\end{aligned} \tag{118}$$

and others $= 0$,

where $W=W_{(n=2,\kappa=\pm1)}$ (see Eq.(34)), and notations of $k_C, W_C, \lambda, s, \kappa$ have been given in Eq.(41). The above results can be expressed in explicit matrix form as follows

$$\{\langle H_1'\rangle_{ii'}\}=\begin{array}{c} (2L^j,L'^{j'})^{m_j,m_j'}: \\ (2s^{1/2})^{1/2} \\ (2s^{1/2})^{-1/2} \\ (2p^{1/2})^{1/2} \\ (2p^{1/2})^{-1/2}\end{array}\begin{array}{c}\begin{array}{cccc}(2s^{1/2})^{1/2} & (2s^{1/2})^{-1/2} & (2p^{1/2})^{1/2} & (2p^{1/2})^{-1/2}\end{array}\\ \left(\begin{array}{cccc} 0 & 0 & 0 & -i\Theta_1 \\ 0 & 0 & -i\Theta_1 & 0 \\ 0 & i\Theta_1 & 0 & 0 \\ i\Theta_1 & 0 & 0 & 0 \end{array}\right)\end{array}, \tag{119}$$

where the matrix row's and column's indices have been labeled explicitly. Compactly, Eq.(119) can be written as follows

$$\{\langle H_1'\rangle_{ii'}\}=\begin{pmatrix} 0 & -i\Theta_1\sigma^1 \\ i\Theta_1\sigma^1 & 0\end{pmatrix}, \quad \text{with} \quad \sigma^1=\begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix}, \tag{120}$$

$$\text{and } \Theta_1=\frac{Q^1Q^0}{4R^2}\frac{W}{3}\sqrt{\frac{k_C^2-W_C^2}{4W_C^2-k_C^2}}\left(\frac{W_C}{k_C}-\frac{k_C}{2W_C}(s+1)\right). \tag{121}$$

2. H_2' matrix elements: H_2' is shown in Eq.(102):

$$H_2'=\frac{Q^1Q^0}{4R^2}\left(i\hbar c\overrightarrow{\frac{\partial}{\partial x^1}}-i\hbar c\overleftarrow{\frac{\partial}{\partial x^1}}\right), \tag{122}$$

$$\text{where } \frac{\partial}{\partial x^1}=\sin\theta\cos\phi\frac{\partial}{\partial r}+\cos\theta\cos\phi\frac{1}{r}\frac{\partial}{\partial\theta}-\frac{\sin\phi}{r\sin\theta}\frac{\partial}{\partial\phi}. \tag{123}$$

By means of straightforward calculations (see Appendix C), we obtain all elements of H_2':

$$\langle H_2'\rangle^{1/2,\,-1/2}_{2s^{1/2},2p^{1/2}}=\langle H_2'\rangle^{-1/2,\,1/2}_{2s^{1/2},2p^{1/2}}=-\langle H_2'\rangle^{1/2,\,-1/2}_{2p^{1/2},2s^{1/2}}=-\langle H_2'\rangle^{-1/2,\,1/2}_{2p^{1/2},2s^{1/2}}$$
$$=i\frac{Q^1Q^0}{2R^2}\frac{\hbar c\lambda}{6\sqrt{4W_C^2-k_C^2}}\left(\frac{k_C^2}{W_C}-2(\frac{1}{s}+1)k_C-W_C+\frac{2}{k_Cs}W_C^2\right)\equiv -i\Theta_2, \tag{124}$$

and others $=0$.

The matrix form of (124) is as follows:

$$\{\langle H_2'\rangle_{ii'}\}=\begin{pmatrix} 0 & -i\Theta_2\sigma^1 \\ i\Theta_2\sigma^1 & 0\end{pmatrix}, \tag{125}$$

where

$$\Theta_2=-\frac{Q^1Q^0}{2R^2}\frac{\hbar c\lambda}{6\sqrt{4W_C^2-k_C^2}}\left(\frac{k_C^2}{W_C}-2(\frac{1}{s}+1)k_C-W_C+\frac{2}{k_Cs}W_C^2\right). \tag{126}$$

3. Since $H'=H_1'+H_2'$ (see Eq.(100)), the matrix form of H' is:

$$\{\langle H'\rangle_{ii'}\}=\{\langle H_1'\rangle_{ii'}\}+\{\langle H_2'\rangle_{ii'}\}=\begin{pmatrix} 0 & -i\Theta\sigma^1 \\ i\Theta\sigma^1 & 0\end{pmatrix}, \tag{127}$$

where

$$\Theta = \Theta_1 + \Theta_2 = \frac{Q^1 Q^0}{4R^2}\frac{W}{3}\sqrt{\frac{k_C^2 - W_C^2}{4W_C^2 - k_C^2}}\left(\frac{W_C}{k_C} - \frac{k_C}{2W_C}(s+1)\right)$$
$$-\frac{Q^1 Q^0}{2R^2}\frac{\hbar c\lambda}{6\sqrt{4W_C^2 - k_C^2}}\left(\frac{k_C^2}{W_C} - 2(\frac{1}{s}+1)k_C - W_C + \frac{2}{k_C s}W_C^2\right) \quad (128)$$

Obviously, $\Theta \propto \mathcal{O}(1/R^2)$.

4. Substituting Eq.(127) into Eq.(105) and Eq.(104), we get the secular equation for eigen value E in the degenerate perturbation calculations:

$$\begin{vmatrix} W-E & 0 & 0 & -i\Theta \\ 0 & W-E & -i\Theta & 0 \\ 0 & i\Theta & W-E & 0 \\ i\Theta & 0 & 0 & W-E \end{vmatrix} = 0. \quad (129)$$

The real energy solutions are

$$W - E = \pm\Theta, \quad \text{or} \quad E^{(+)} = W + \Theta, \quad E^{(-)} = W - \Theta, \quad (130)$$

and hence

$$(\Delta E)_{(2s^{1/2}-2p^{1/2})} \equiv E^{(+)} - E^{(-)} = 2\Theta$$
$$= \frac{Q^1 Q^0}{R^2}\left[\frac{W}{6}\sqrt{\frac{k_C^2 - W_C^2}{4W_C^2 - k_C^2}}\left(\frac{W_C}{k_C} - \frac{k_C}{2W_C}(s+1)\right)\right.$$
$$\left.-\frac{\hbar c\lambda}{6\sqrt{4W_C^2 - k_C^2}}\left(\frac{k_C^2}{W_C} - 2(\frac{1}{s}+1)k_C - W_C + \frac{2}{k_C s}W_C^2\right)\right], \quad (131)$$

which is $\mathcal{O}(1/R^2)$ and the desired expression of energy level splitting of $(2s^{1/2} - 2p^{1/2})$ due to H'. Eq.(131) represents an important effect of dS-SR to one-electron atom, and it is the main result of this paper.

5. The eigenstates with eigenvalues $E^{(\pm)}$: Generally, the eigenstates of H' are

$$|E^{(\pm)}\rangle = C_1^{(\pm)}|2s^{1/2}\rangle^{1/2} + C_2^{(\pm)}|2s^{1/2}\rangle^{-1/2} + C_3^{(\pm)}|2p^{1/2}\rangle^{1/2} + C_4^{(\pm)}|2p^{1/2}\rangle^{-1/2}, \quad (132)$$

where $\{|2s^{1/2}\rangle^{m_j},\ |2s^{1/2}\rangle^{m_j}\} \in |nL^j\rangle^{m_j}$, and $C_i^{(\pm)}$ $(i = 1,2,3,4)$ satisfy the following eigen-equation corresponding to Eq.(129):

$$\begin{pmatrix} W & 0 & 0 & -i\Theta \\ 0 & W & -i\Theta & 0 \\ 0 & i\Theta & W & 0 \\ i\Theta & 0 & 0 & W \end{pmatrix}\begin{pmatrix} C_1^{(\pm)} \\ C_2^{(\pm)} \\ C_3^{(\pm)} \\ C_4^{(\pm)} \end{pmatrix} = E^{(\pm)}\begin{pmatrix} C_1^{(\pm)} \\ C_2^{(\pm)} \\ C_3^{(\pm)} \\ C_4^{(\pm)} \end{pmatrix}. \quad (133)$$

Substituting (130) into (133), we get the eigenstates with eigenvalues $E^{(\pm)}$:

$$|E^{(+)}\rangle = \frac{1}{2}(|2s^{1/2}\rangle^{1/2} + |2s^{1/2}\rangle^{-1/2} + i|2p^{1/2}\rangle^{1/2} + i|2p^{1/2}\rangle^{-1/2}), \quad (134)$$
$$|E^{(-)}\rangle = \frac{1}{2}(|2s^{1/2}\rangle^{1/2} + |2s^{1/2}\rangle^{-1/2} - i|2p^{1/2}\rangle^{1/2} - i|2p^{1/2}\rangle^{-1/2}), \quad (135)$$

which satisfy $\langle E^{(+)}|E^{(+)}\rangle = \langle E^{(-)}|E^{(-)}\rangle = 1$, and $\langle \mathrm{E}^{(-)}|\mathrm{E}^{(+)}\rangle = 0$.

6. Numerical discussions: In order to getting comprehensive indications to Eq.(131), we now discuss $(\Delta E)_{(2s^{1/2}-2p^{1/2})} \equiv E^{(+)} - E^{(-)} = 2\Theta$ numerically. To Hydrogen's $2s^{1/2}$- and $2p^{1/2}$-states, $\mu \simeq m_e = 510998.910\ eV/c^2$, $Z = 1$, $n = 2$, $\kappa = \pm 1$. Substituting them into Eq.(34), we get $W = 510995.51\ eV/c^2$. And then, by Eq.(41), we further have k_C, W_C, λ and s. Inserting all of them into Eq.(131), we obtain

$$\begin{aligned} \Delta E(z) \equiv (\Delta E)_{(2s^{1/2}-2p^{1/2})} &= \frac{Q^1(z)Q^0(z)}{R^2} \times 358.826\ eV \\ &= \frac{Q^1(z)Q^0(z)}{R^2} \times 8.36 \times 10^7\ (Lamb\ shift), \end{aligned} \tag{136}$$

where $Q^0(z) \equiv ct(z)$ and $Q^1(z)$ have been given in Eqs.(20) and (24) respectively (see also FIG.2 and FIG.3). In the expression of function $\Delta E(z)$ of Eq.(136), when z were fixed, the only unknown number is R which is the universal parameter of dS-SR. Therefore, it is expected that R could be determined through accurate enough observations of the level spectrum shifts of atoms on distant galaxy. The curves of $\Delta E(z)$ of Eq.(136) with $|R| = \{0.5 \times 10^5 Gly,\ 10^5 Gly,\ 2 \times 10^5 Gly\}$ are shown in FIG.6. It is essential that FIG.6 shows that when $z \geq 1$, $\Delta E(z) >> 1$ Lamb shift. This indicates that comparing with the usual QED's hyperfine structure effects (i.e., the Lamb shift measured in the laboratory), the dS-SR QM fine structure effects are dominating for the splitting between $2s^{1/2}$- and $2p^{1/2}$- states of Hydrogen atom on distant galaxy. In the TABLE I, the $\Delta E(z)$ for $|R| = \{10^3 Gly,\ 10^4 Gly,\ 10^5 Gly\}$ and $z = \{1,\ 2\}$ is listed. It is learned that $\Delta E(z) >> 1$ Lamb shift to all cases too.

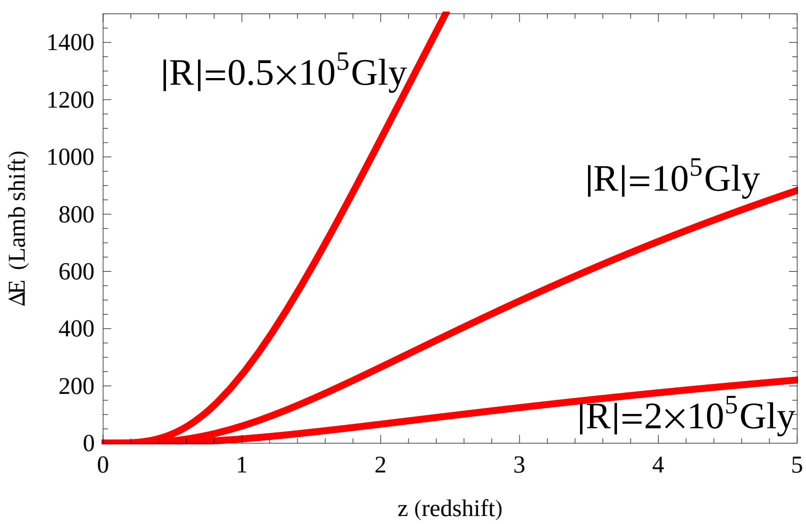

Figure 6: Functions $\Delta E(z)$ of Eq.(131) with $|R| = 0.5 \times 10^5 Gly,\ 10^5 Gly,\ 2 \times 10^5 Gly$ are shown. The unit of the energy splitting $\Delta E(z)$ is (Lamb shift)$\simeq 4.3 \times 10^{-6} eV$.

9 Summary and discussions

In this paper, we have solved the de Sitter special relativistic Dirac equation of Hydrogen in the Earth-QSO framework reference by means of the adiabatic approach and the quasi-stationary perturbation calculations of QM. Hydrogen atoms are located on the light cone of the Universe. FRW metric and ΛCDM cosmological model are used to discuss this issue. To

Table 1: The energy level splitting of Hydrogen's $(2s^{1/2} - 2p^{1/2})$, $\Delta E(z) \equiv (\Delta E)_{(2s^{1/2}-2p^{1/2})}$ (see Eqs. (131) (136)): R is the radius of de Sitter pseudo-sphere in dS-SR, which is a universal parameter in the theory and $|R| > R_H \equiv 13.7Gly$(Horizon of the Universe). $Gly \equiv 10^9$ *light years*. z is the red shift. (Lamb shift)$\simeq 4.3 \times 10^{-6} eV$.

$\vert R\vert$	$10^3 Gly$		$10^4 Gly$		$10^5 Gly$	
z	1	2	1	2	1	2
$\Delta E(z)$ (eV)	2.6	11	2.6×10^{-2}	0.11	2.6×10^{-4}	1.1×10^{-3}
(Lamb shift)	6×10^5	2.7×10^6	5977	26541	59.77	265.41

the atom, effects of de Sitter space-time geometry described by Beltrami metric are taken into account. The dS-SR Dirac equation of Hydrogen turns out to be a time dependent quantum Hamiltonian system. We have provided an explicit calculation to examine whether the adiabatic approach to deal with this time-dependent system is eligible. Since the radius of de Sitter sphere $|R|$ is cosmologically large, it makes the time-evolution of the system is so slow that the adiabatic approximation legitimately works with high accuracy. Based the dS-SR Dirac equation's solutions up to $\mathcal{O}(1/R^2)$, some remarkable effects of dS-SR atom physics are revealed:

1. The fundamental physics constants variate adiabatically along with cosmologic time in dS-SR quantum mechanics framework. As is well known that the quantum evolution in the time-dependent quantum mechanics has been widely accepted and studied during past several decades. It is remarkable that the time-variations of μ $(or\ m_e)$, $\hbar$ and e (see Eqs. (77) (78) (79)) belong to such quantum evolution effects.

2. The fine-structure constant $\alpha \equiv e^2/(\hbar c)$ keeps invariant along with time up to $\mathcal{O}(1/R^2)$ [26]. In the expression of α, the e^2's time-variation and $\hbar$'s are canceled each other. However, whether or not this cancelation mechanism works to the next order $\mathcal{O}(1/R^4)$ remains to be open so far.

3. $(2s^{1/2} - 2p^{1/2})$-splitting due to dS-SR Dirac QM effects: Distinguishing from E-SR Dirac QM theory of Hydrogen atom, the degeneracy of $(2s^{1/2} - 2p^{1/2})$ is broken in dS-SR QM. By means of the quasi-stationary perturbation theory, the $(2s^{1/2} - 2p^{1/2})$-splitting $\Delta E(z)$ has been calculated analytically, which belongs to $\mathcal{O}(1/R^2)$-physics of dS-SR QM. Numerically, we found that when $|R| \simeq \{10^3 Gly,\ 10^4 Gly,\ 10^5 Gly\ \}$ (note the Universal horizon $R_H \simeq 13.7Gly << |R|$), and $z \simeq \{1,\ \text{or}\ 2\}$, we have $\Delta E(z) >> 1(\text{Lamb shift})$. This indicate that for this case the hyperfine structure effects due to QED could be ignored, and the dS-SR fine structure effects are dominant. Therefore, we suggest that this effect could be used to determine the universal constant R in dS-SR, and be thought as a test of new physics beyond E-SR.

Finally, we address again that the dS-SR is a natural extension of E-SR. What we achieved in this paper are that we revealed new effects in one-electron atom physics, which are beyond E-SR and hence can be used to recognize dS-SR.

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Appendix A: Electric Coulomb Law in QSO-Light-Cone Space

Let's derive (62). The action for deriving electrostatic potential of proton located at $Q^\mu = \{Q^0 = ct,\ Q^1,\ Q^2 = 0,\ Q^3 = 0\}$ within background space-time metric $g_{\mu\nu} \equiv B_{\mu\nu}(Q)$ of eq.(16) in the Gaussian system of units reads

$$S = -\frac{1}{16\pi c}\int d^4L\sqrt{-g}F_{\mu\nu}F^{\mu\nu} - \frac{e}{c}\int d^4L\sqrt{-g}j^\mu A_\mu, \tag{137}$$

where $g = \det(B_{\mu\nu})$, $F_{\mu\nu} = \frac{\partial A_\nu}{\partial L^\mu} - \frac{\partial A_\mu}{\partial L^\nu}$ and $j^\mu \equiv \{j^0 = c\rho_{proton}/\sqrt{B_{00}},\ \mathbf{j}\}$ is 4-current density vector of proton (see, e.g, Ref. [27]: *Chapter 4; Chapter 10, Eq.(90.3)*). The explicit matrix-expressions for $B_{\mu\nu}(Q)$ and $B^{\mu\nu}(Q)$ up to $\mathcal{O}(1/R^2)$ are follows:

$$B_{\mu\nu}(Q) = \begin{pmatrix} 1+\frac{2(Q^0)^2-(Q^1)^2}{R^2} & -\frac{Q^0Q^1}{R^2} & 0 & 0 \\ -\frac{Q^1Q^0}{R^2} & -1+\frac{2(Q^1)^2-(Q^0)^2}{R^2} & 0 & 0 \\ 0 & 0 & -1-\frac{(Q^0)^2-(Q^1)^2}{R^2} & 0 \\ 0 & 0 & 0 & -1-\frac{(Q^0)^2-(Q^1)^2}{R^2} \end{pmatrix},$$

$$B^{\mu\nu}(Q) = \begin{pmatrix} 1-\frac{2(Q^0)^2-(Q^1)^2}{R^2} & \frac{Q^0Q^1}{R^2} & 0 & 0 \\ \frac{Q^1Q^0}{R^2} & -1-\frac{2(Q^1)^2-(Q^0)^2}{R^2} & 0 & 0 \\ 0 & 0 & -1+\frac{(Q^0)^2-(Q^1)^2}{R^2} & 0 \\ 0 & 0 & 0 & -1+\frac{(Q^0)^2-(Q^1)^2}{R^2} \end{pmatrix}.$$

Making space-time variable change of $L^\mu \to L^\mu - Q^\mu \equiv x^\mu = \{x^0 = ct_L - ct,\ x^i = L^i - Q^i\}$, we have action S as

$$\begin{aligned} S = -\frac{1}{16\pi c} & \int d^4x\sqrt{-\det(B_{\mu\nu}(Q))}F_{\mu\nu}F^{\mu\nu} - \frac{e}{c}\int d^4x\sqrt{-\det(B_{\mu\nu}(Q))}j^\mu A_\mu \\ = \Big(-\frac{1}{16\pi c} & \quad B_{\mu\lambda}(Q)B_{\nu\rho}(Q)\int d^4x F^{\mu\nu}(x)F^{\lambda\rho}(x) \\ & -\frac{e}{c}\int d^4x j^\mu(x)A_\mu(x)\Big)\sqrt{-\det(B_{\mu\nu}(Q))}, \end{aligned} \tag{138}$$

and the equation of motion $\delta S/\delta A_\mu(x) = 0$ as follows (see, e.g, [27], Eq. (90.6), *pp257*)

$$\partial_\nu F^{\mu\nu} = B^{\nu\lambda}\partial_\nu F^\mu{}_\lambda = -\frac{4\pi}{c}j^\mu. \tag{139}$$

In Beltrami space, $A^\mu = \{\phi_B,\ \mathbf{A}\}$ (see, e.g., [27], eq.(16.2) in *pp. 45*) and 4-charge current $j^\mu = \{c\rho_{proton}/\sqrt{B_{00}},\ \mathbf{j}\}$. According to the expression of charge density in curved space in

Ref. [27], (*pp.256, Eq. (90.4)*), $\rho_{proton} \equiv \rho_B = \frac{e}{\sqrt{\gamma}}\delta^{(3)}(\mathbf{x})$ and $\mathbf{j} = 0$, where

$$\gamma = \det(\gamma_{ij}), \tag{140}$$

$$\begin{aligned} dl^2 = \gamma_{ij}dx^i dx^j &= \left(-g_{ij} + \frac{g_{0i}g_{j0}}{g_{00}}\right) dx^i dx^j \quad (see\ eq.(84.7)\ in\ Ref.[37]) \\ &= \left(-B_{ij} + \frac{B_{0i}B_{j0}}{B_{00}}\right) dx^i dx^j \end{aligned} \tag{141}$$

Noting $B_{01} = B_{10} = -\frac{C^2t^2}{R^2}$, $B_{00} \sim 1$, and $B_{01}B_{10} \simeq \mathcal{O}(1/R^4) \sim 0$, we have

$$\sqrt{\gamma} \equiv \sqrt{\det(\gamma_{ij})} \simeq \sqrt{-\det(B_{ij})}, \tag{142}$$

and hence

$$\rho_{proton} \equiv \rho_B = \frac{e\delta^{(3)}(\mathbf{x})}{\sqrt{-\det(B_{ij})}}, \quad \mathbf{j} = 0. \tag{143}$$

1. When $\mu = 0$ in Eq.(139), we have the Coulomb's law (60), i.e.,

$$\begin{aligned} -B^{ij}(Q)\partial_i\partial_j\phi_B(x) &= \left[\left(1 - \frac{(Q^0)^2 - (Q^1)^2}{R^2}\right)\nabla^2 + \frac{(Q^1)^2}{R^2}\frac{\partial^2}{\partial(x^1)^2}\right]\phi_B(x) = -\frac{4\pi}{c}j^0 \\ &= \frac{-4\pi e}{\sqrt{-\det(B_{ij}(Q))B_{00}(Q)}}\delta^{(3)}(\mathbf{x}), \end{aligned} \tag{144}$$

   where $B^{ij}(Q) = \eta^{ij} + \frac{(Q^0)^2 - 2(Q^1)^2}{R^2}\delta_{i1}\delta_{j1} + \frac{(Q^0)^2-(Q^1)^2}{R^2}\delta_{i2}\delta_{j2} + \frac{(Q^0)^2-(Q^1)^2}{R^2}\delta_{i3}\delta_{j3} + \mathcal{O}(R^{-4})$ has been used, and B_{ij} were given in (16). Expanding (144), we have

$$\begin{aligned} &\left[\frac{\partial^2}{\partial(x^1/[1+\frac{(Q^1)^2}{2R^2}])^2} + \frac{\partial^2}{\partial(x^2)^2} + \frac{\partial^2}{\partial(x^3)^2}\right]\phi_B(x) = -4\pi\left(1 - \frac{3(Q^0)^2 - 4(Q^1)^2}{2R^2}\right) \\ &\qquad \times \left(1 - \frac{2(Q^0)^2 - (Q^1)^2}{2R^2}\right)\left(1 + \frac{(Q^0)^2 - (Q^1)^2}{R^2}\right) e\delta(x^1)\delta(x^2)\delta(x^3) \\ &\qquad = -4\pi e\left(1 - \frac{3[(Q^0)^2 - (Q^1)^2]}{2R^2}\right)\delta(x^1)\delta(x^2)\delta(x^3). \end{aligned}$$

   Noting $\delta(x^1) = \delta(x^1/[1+(Q^1)^2/2R^2])(1-(Q^1)^2/2R^2)$, we rewrite above equation as follows

$$\begin{aligned} &\left[\frac{\partial^2}{\partial(x^1/[1+\frac{(Q^1)^2}{2R^2}])^2} + \frac{\partial^2}{\partial(x^2)^2} + \frac{\partial^2}{\partial(x^3)^2}\right]\phi_B(x) = -4\pi e\left(1 - \frac{3[(Q^0)^2 - (Q^1)^2]}{2R^2}\right) \\ &\qquad \times \left(1 - \frac{(Q^1)^2}{2R^2}\right)\delta(x^1/[1+\frac{(Q^1)^2}{2R^2}])\delta(x^2)\delta(x^3) \\ &\qquad = -4\pi e\left(1 - \frac{3(Q^0)^2 - 2(Q^1)^2}{2R^2}\right)\delta(x^1/[1+\frac{(Q^1)^2}{2R^2}])\delta(x^2)\delta(x^3). \end{aligned}$$

   Setting

$$\tilde{x}^1 \equiv x^1/[1+\frac{(Q^1)^2}{2R^2}], \tag{145}$$

   the above equation becomes

$$\left[\frac{\partial^2}{\partial(\tilde{x}^1)^2} + \frac{\partial^2}{\partial(x^2)^2} + \frac{\partial^2}{\partial(x^3)^2}\right]\phi_B(x) = -4\pi e\left(1 - \frac{3(Q^0)^2 - 2(Q^1)^2}{2R^2}\right)\delta(\tilde{x}^1)\delta(x^2)\delta(x^3). \tag{146}$$

Then the solution is $\phi_B(x) = \left(1 - \frac{3(Q^0)^2 - 2(Q^1)^2}{2R^2}\right) e/r_B$ with

$$\begin{aligned} r_B &= \sqrt{(\tilde{x}^1)^2 + (x^2)^2 + (x^3)^2} \\ &= \left((1 - \frac{2(Q^1)^2 - (Q^0)^2}{2R^2})^2 (x^1)^2 + (x^2)^2 + (x^3)^2\right)^{1/2}. \end{aligned}$$

Therefore, we have

$$\phi_B = \left(1 - \frac{3(Q^0)^2 - 2(Q^1)^2}{2R^2}\right) \frac{e}{r_B}, \tag{147}$$

which is the scalar potential in Eq.(62) in the text.

2. When $\mu = i$ ($i = 1, 2, 3$) in Eq.(139), we have

$$\partial^i \partial_\mu A^\mu - B^{\mu\nu} \partial_\mu \partial_\nu A^i = -\frac{4\pi}{c} j^i = 0. \tag{148}$$

By means of the gauge condition

$$\partial_\mu A^\mu = 0, \tag{149}$$

we have

$$B^{\mu\nu} \partial_\mu \partial_\nu A^i = 0. \tag{150}$$

Then

$$A^i = 0 \tag{151}$$

is a solution that satisfies the gauge condition (149) (noting $\partial_0 A^0 = \frac{\partial}{\partial x^0} \phi_B(r_B) = 0$ due to $\frac{\partial Q^0}{\partial x^0} = \frac{\partial Q^0}{\partial L^0} = 0$). Eq.(151) is the vector potential in Eq.(62) in the text.

Appendix B: Adiabatic approximative wave functions in $\mathcal{SR}_{cR}$-Dirac equation of hydrogen

Now we derive the wave function of (85) in the text. We start with eq.(82), i.e.

$$i\hbar \partial_t \psi = H(t)\psi = [H_0(r_B, \hbar, \mu, e) + H_0'(t)]\psi, \tag{152}$$

where

$$\begin{aligned} H(t) &= H_0(r_B, \hbar, \mu, e) + H_0'(t), & (153) \\ H_0(r_B, \hbar, \mu, e) &= -i\hbar c \vec{\alpha} \cdot \nabla_B + \mu c^2 \beta - \frac{e^2}{r_B} \quad (\textit{see eq.}(33)) & (154) \\ H_0'(t) &= -\left(\frac{c^2 t^2}{2R^2}\right) H_0(r_B, \hbar, 3\mu, e). & (155) \end{aligned}$$

Suppose the modification of $H(t)$ along with the time change is sufficiently slow, the system could be quasi-stationary in any instant θ. Then, in the Shrödinger picture, the quasi-stationary equation of $H(\theta)$

$$H(\theta) U_n(\mathbf{x}, \theta) = W_n(\theta) U_n(\mathbf{x}, \theta) \tag{156}$$

can be solved. By (153) (154) (155) and $t \to \theta$, the solutions are as follows (similar to eq.(34) in text)

$$\begin{aligned} W_n(\theta) \equiv W_{n,\kappa}(\theta) &= \mu_\theta c^2 \left(1 + \frac{\alpha^2}{(n - |\kappa| + s)^2}\right)^{-1/2} \qquad (157) \\ & \alpha \equiv \frac{e_\theta^2}{\hbar_\theta c} \simeq \frac{e^2}{\hbar c} + \mathcal{O}(\frac{c^4\theta^4}{R^4}), \quad |\kappa| = (j + 1/2) = 1,\ 2,\ 3\ \cdots \\ & s = \sqrt{\kappa^2 - \alpha^2}, \qquad n = 1,\ 2,\ 3\ \cdots . \end{aligned}$$

where (see (77), (78), (79) in text)

$$\hbar_\theta = \left(1 - \frac{c^2\theta^2}{2R^2}\right)\hbar, \tag{158}$$

$$\mu_\theta = \left(1 - \frac{(Q^1(\theta))^2}{2R^2}\right)\mu, \tag{159}$$

$$e_\theta = \left(1 - \frac{c^2\theta^2}{4R^2}\right)e, \tag{160}$$

The complete set of commutative observable is $\{H,\ \kappa,\ \mathbf{j}^2,\ j_z\}$, so that we have

$$U_n(\mathbf{x}, \theta) = \psi_{n,\kappa,j,j_z}(\mathbf{r}_B, \hbar_\theta, \mu_\theta, e_\theta), \tag{161}$$

where $\mathbf{j} = \mathbf{L} + \frac{\hbar}{2}\mathbf{\Sigma},\ \hbar\kappa = \beta(\mathbf{\Sigma}\cdot\mathbf{L} + \hbar)$. $[U_n(\mathbf{x}, \theta)]$ is complete set and satisfies

$$\int d^3x U_n(\mathbf{x}, \theta) U_m^*(\mathbf{x}, \theta) = \delta_{mn}, \qquad n = \{n_r, K, j, j_z\}. \tag{162}$$

Thus, the solution of time-dependent Shrödinger equation (or Dirac equation) (152) can expanded as follows

$$\psi(\mathbf{x}, t) = \sum_n C_n(t) U_n(\mathbf{x}, t) \exp\left[-i\int_0^t \omega_n(\theta) d\theta\right], \quad \omega_n(\theta) = \frac{W_n(\theta)}{\hbar}. \tag{163}$$

Substituting (163) into (152), we have

$$i\hbar \sum_n (\dot{C}_n U_n + C_n \dot{U}_n) \exp\left[-i\int_0^t \omega_n(\theta) d\theta\right] = 0. \tag{164}$$

By multiplying $U_m^* \exp\left[i\int_0^t \omega_m(\theta) d\theta\right]$ to both sides of eq.(164), and doing integral to $\mathbf{x}$ by using (162), we have

$$\begin{aligned} \dot{C}_m + C_m \int d^3x U_m^* \dot{U}_m \quad &+ \quad \sum_n{}' C_n \int d^3x U_m^* \dot{U}_n \exp\left[-i\int_0^t (\omega_n - \omega_m) d\theta\right] = 0, \qquad (165) \\ m \quad &= 1, 2, 3, \cdots \end{aligned}$$

where $\sum_n'$ means that $n \neq m$ in the summation over n. Noting (162), we have

$$\int \dot{U}_m^* U_m d^3x + \int U_m^* \dot{U}_m d^3x = 0, \tag{166}$$

and hence

$$\int U_m^* \dot{U}_m d^3x = i\beta \tag{167}$$

is purely imaginary number. Denoting

$$\alpha_{mn} = \int U_m^* \dot{U}_n d^3x, \quad \text{and} \quad \omega_{nm} = \omega_n - \omega_m, \tag{168}$$

then eq.(165) becomes

$$\dot{C}_m + i\beta C_m + \sum_n{}' C_n \alpha_{mn} \exp\left[-i\int_0^t \omega_{nm} d\theta\right] = 0. \qquad m \ = 1,2,3,\cdots \tag{169}$$

To further simplify it, we set

$$V_n(\mathbf{x},t) = U_n(\mathbf{x},t)\exp\left[-i\int_0^t \beta_n(\theta)d\theta\right], \tag{170}$$

then

$$\psi(\mathbf{x},t) = \sum_n C_n^{'}(t) V_n(\mathbf{x},t)\exp\left[-i\int_0^t \omega_n(\theta)d\theta\right], \tag{171}$$

where $C_n^{'}(t) = C_n(t)\exp\left[i\int_0^t \beta_n(\theta)d\theta\right]$, and

$$\dot{C}_m^{'}(t) = [\dot{C}_m + i\beta_m C_m(t)]\exp\left(i\int_0^t \beta_n(\theta)d\theta\right) \tag{172}$$

Substituting (172) into (169), we finally get

$$\dot{C}_m^{'} + \sum_n{}' C_n^{'} \alpha_{mn} \exp\left[-i\int_0^t \omega_{nm}^{\prime} d\theta\right] = 0. \qquad m \ = 1,2,3,\cdots \tag{173}$$

where

$$\omega_{mn}^{\prime} = \omega_n^{\prime} - \omega_m^{\prime}, \quad \omega_n^{\prime} = \frac{1}{\hbar}W_n + \beta_n. \tag{174}$$

Now let's solve (173). Firstly, we derive α_{mn}. By (156), we have

$$\frac{\partial H}{\partial t}U_n + H\dot{U}_n = \dot{W}_n U_n + W_n \dot{U}_n. \tag{175}$$

By multiplying U_m^* and doing integral over $\mathbf{x}$, we have

$$\begin{aligned} \int U_m^* \dot{H} U_n d^3x \ &+ \ \int U_m^* H \dot{U}_n d^3x = W_n \int U_m^* \dot{U}_n d^3x \\ \text{i.e.,} \qquad \dot{H}_{mn} + W_m \alpha_{mn} \ &= \ W_n \alpha_{mn}, \end{aligned} \tag{176}$$

so that

$$\alpha_{mn} = \int U_m^* \dot{U}_n d^3x = \frac{1}{W_n - W_m}\dot{H}_{mn}, \quad m \neq n. \tag{177}$$

Therefore eq.(173) becomes

$$\dot{C}_m^{'} + \sum_n{}' C_n^{'} \frac{\dot{H}_{mn}\exp\left(-i\int_0^t \omega_{nm}^{\prime} d\theta\right)}{\hbar\omega_{nm}} = 0. \quad m = 1,2,3,\cdots \tag{178}$$

Suppose in the initial time the system is in s-state, i.e., $C_n(0) = C_n^{'}(0) = \delta_{ns}$. To adiabatic process, $\dot{H}(t) \to 0$, then the 0-order approximative solution of eq.(178) is

$$[C_m^{'}(t)]_0 = \delta_{ms}. \tag{179}$$

Substituting (179) into (178), we get the first order correction to the approximation

$$[\dot{C}_m^{'}]_1 = \frac{-\dot{H}_{ms}}{\hbar\omega_{ms}} \exp\left(-i\int_0^t \omega'_{ms} d\theta\right) = 0, \qquad m \neq s. \tag{180}$$

Since the dependent on time t of $U_n(t)$ is weak for adiabatic process, eq.(167) indicates β_n is small, and by (174), we have $\omega'_{ms} \approx \omega_{ms}$. Then, from (180), the first order correction to the solution is

$$[C_m^{'}]_1 = \frac{\dot{H}_{ms}}{i\hbar\omega_{ms}}\left(e^{i\omega_{ms}t} - 1\right), \qquad m \neq s. \tag{181}$$

Substituting (180) (181) into (171) and neglecting β_n, we get the wave function as follows

$$\psi(\mathbf{x}, t) \simeq U_s(\mathbf{x}, t) e^{-i\frac{W_s t}{\hbar}} + \sum_{m\neq s} \frac{\dot{H}_{ms}}{i\hbar\omega_{ms}}\left(e^{i\omega_{ms}t} - 1\right) U_m(\mathbf{x}, t) e^{\left(-i\int_0^t \frac{W_m(\theta)}{\hbar} d\theta\right)}. \tag{182}$$

By using eqs.(161), (159), (158), (160), we finally obtain the desired results

$$\psi(t) \simeq \psi_s(\mathbf{r}_B, \hbar_t, \mu_t, e_t) e^{-i\frac{W_s}{\hbar}t} + \sum_{m\neq s} \frac{\dot{H}'(t)_{ms}}{i\hbar\omega_{ms}^2}\left(e^{i\omega_{ms}t} - 1\right) \psi_m(\mathbf{r}_B, \hbar_t, \mu_t, e_t) e^{\left(-i\int_0^t \frac{W_m(\theta)}{\hbar} d\theta\right)}, \tag{183}$$

where

$$\hbar_t = \left(1 - \frac{c^2t^2}{2R^2}\right)\hbar, \tag{184}$$

$$\mu_t = \left(1 - \frac{(Q^1(t))^2}{2R^2}\right)\mu, \tag{185}$$

$$e_t = \left(1 - \frac{c^2t^2}{4R^2}\right)e, \tag{186}$$

(184) (185) and (186) are the equations (77), (78) and (79) in the text. Eq.(183) is just Eq.(85) in the text.

Appendix C: Calculations of elements of the perturbation Hamiltonian H$'$-matric in $(2s^{1/2}$-$2p^{1/2})$-Hilbert space

Now we derive Eq.(118) and Eq.(124). We start with the dS-SR Dirac spectrum equation of Hydrogen, which has been shown in Eqs. (98)-(102) in the text:

$$H_{(dS-SR)}\psi = (\ H_0(r, \hbar_t, \mu_t, e_t) + H'\ )\psi = E\psi, \tag{187}$$

where

$$H_0(r, \hbar_t, \mu_t, e_t) = -i\hbar_t c\vec{\alpha}\cdot\nabla + \mu_t c^2\beta - \frac{e_t^2}{r}, \tag{188}$$

$$H' = \frac{1}{2}(H'^{\dagger} + H') \equiv H'_1 + H'_2, \tag{189}$$

where

$$H_1' = -\frac{Q^1Q^0}{4R^2}\left(\alpha^1 H_0(r,\hbar,\mu,e) + H_0(r,\hbar,\mu,e)\alpha^1\right), \tag{190}$$

$$H_2' = \frac{Q^1Q^0}{4R^2}\left(i\hbar c\overrightarrow{\frac{\partial}{\partial x^1}} - i\hbar c\overleftarrow{\frac{\partial}{\partial x^1}}\right). \tag{191}$$

The definition of H'-elements in the H_0-eigenstate space, $\langle H'\rangle^{m_j,\,m_j'}_{2L^{1/2},2L'^{1/2}}$, has been given in Eqs.(115) (116). The eigen values and eigen states of H_0 are given in the section IV.

(I) H_1'-matrix elements:

1. $\langle H_1'\rangle^{1/2,\,-1/2}_{2s^{1/2},2p^{1/2}}$:

$$\langle H_1'\rangle^{1/2,\,-1/2}_{2s^{1/2},2p^{1/2}} = \int drr^2\int d\Omega\;\psi^{1/2\dagger}_{(2s)j=1/2}(\mathbf{r})\;H_1'\;\psi^{-1/2}_{(2p)j=1/2}(\mathbf{r}) = -\frac{Q^1Q^0}{4R^2}W\langle(2s^{1/2})^{1/2}|\alpha^1|(2p^{1/2})^{-1/2}\rangle$$

$$= -\frac{Q^1Q^0}{4R^2}W\int drr^2\int d\Omega\Big(g_{(2s^{1/2})}(r)\chi^{1/2\dagger}_{\kappa}(\hat{\mathbf{r}})_{(2s^{1/2})}, -if_{(2s^{1/2})}(r)\chi^{1/2\dagger}_{-\kappa}(\hat{\mathbf{r}})_{(2s^{1/2})}\Big)\begin{pmatrix}0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix}$$

$$\times\begin{pmatrix}g_{(2p^{1/2})}(r)\chi^{-1/2}_{\kappa}(\hat{\mathbf{r}})_{(2p^{1/2})}\\ if_{(2p^{1/2})}(r)\chi^{-1/2}_{-\kappa}(\hat{\mathbf{r}})_{(2p^{1/2})}\end{pmatrix}$$

$$= -i\frac{Q^1Q^0}{4R^2}W\int_0^\infty drr^2\Big\{g_{(2s^{1/2})}(r)f_{(2p^{1/2})}(r)\int d\Omega\chi^{1/2\dagger}_{\kappa}(\hat{\mathbf{r}})_{(2s^{1/2})}\sigma^1\chi^{-1/2}_{-\kappa}(\hat{\mathbf{r}})_{(2p^{1/2})}$$
$$-f_{(2s^{1/2})}(r)g_{(2p^{1/2})}(r)\int d\Omega\chi^{1/2\dagger}_{-\kappa}(\hat{\mathbf{r}})_{(2s^{1/2})}\sigma^1\chi^{-1/2}_{\kappa}(\hat{\mathbf{r}})_{(2p^{1/2})}\Big\} \tag{192}$$

where $W = W_{(n=2,\kappa=\pm1)}$, and the explicit expressions of $2s^{1/2}$- and $2p^{1/2}$ -wave functions of $\{g_{(2s^{1/2})}(r),\; f_{(2s^{1/2})}(r),\; g_{(2p^{1/2})}(r),\; f_{(2p^{1/2})}(r),\; \chi^{\pm1/2}_{\pm\kappa}(\hat{\mathbf{r}})_{(2s^{1/2})},\; \chi^{\pm1/2}_{\pm\kappa}(\hat{\mathbf{r}})_{(2p^{1/2})}\}$ are given in Eqs.(43)$-$(52) in text. From them, we have

$$\int d\Omega\chi^{1/2\dagger}_{\kappa}(\hat{\mathbf{r}})_{(2s^{1/2})}\sigma^1\chi^{-1/2}_{-\kappa}(\hat{\mathbf{r}})_{(2p^{1/2})}$$
$$= \int_0^{2\pi}d\phi\int_0^{\pi}d\theta\sin\theta\left(Y_0^0,0\right)\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\sqrt{\frac{2}{3}}\cos\theta Y_1^{-1}(\theta\phi) - \sqrt{\frac{1}{3}}\sin\theta e^{-i\phi}Y_1^0(\theta\phi)\\ \sqrt{\frac{2}{3}}\sin\theta e^{i\phi}Y_1^{-1}(\theta\phi) - \sqrt{\frac{1}{3}}\cos\theta Y_1^0(\theta\phi)\end{pmatrix}$$
$$= \frac{1}{2}\int_0^{\pi}d\theta\sin\theta(\sin\theta\cos\theta + \cos^2\theta) = \frac{1}{3}; \tag{193}$$

$$\int d\Omega\chi^{1/2\dagger}_{-\kappa}(\hat{\mathbf{r}})_{(2s^{1/2})}\sigma^1\chi^{-1/2}_{\kappa}(\hat{\mathbf{r}})_{(2p^{1/2})}$$
$$= \int_0^{2\pi}d\phi\int_0^{\pi}d\theta\sin\theta\left(-\cos\theta Y_0^0, -\sin\theta e^{-i\phi}Y_0^0\right)\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}-\sqrt{\frac{2}{3}}Y_1^{-1}(\theta\phi)\\ \sqrt{\frac{1}{3}}Y_1^0(\theta\phi)\end{pmatrix}$$
$$= -\frac{1}{2}\int_0^{\pi}d\theta\sin\theta\cos^2\theta = -\frac{1}{3}, \tag{194}$$

where $Y_1^0(\theta\phi) = \sqrt{\frac{3}{4\pi}}\cos\theta,\; Y_1^1(\theta\phi) = -\sqrt{\frac{3}{8\pi}}e^{i\phi}\sin\theta,\; Y_1^{-1}(\theta\phi) = \sqrt{\frac{3}{8\pi}}e^{-i\phi}\sin\theta$ and $Y_0^0(\theta\phi) = \sqrt{\frac{1}{4\pi}}$ have been used. Substituting Eqs.(193) (194) into (192), we get

$$\langle H_1'\rangle^{1/2,\,-1/2}_{2s^{1/2},2p^{1/2}} = -i\frac{Q^1Q^0}{4R^2}\frac{W}{3}\int_0^\infty drr^2\left\{g_{(2s^{1/2})}(r)f_{(2p^{1/2})}(r) + f_{(2s^{1/2})}(r)g_{(2p^{1/2})}(r)\right\} \tag{195}$$

Inserting the explicit expressions of radial wave functions $g_{(2s^{1/2})}(r)$, $f_{(2s^{1/2})}(r)$, and $g_{(2p^{1/2})}(r)$, $f_{(2p^{1/2})}(r)$ (i.e. Eqs.(44), (45), (49), (50) in text) into the integral in Eq.(195), and accomplishing the calculations, we have

$$\int_0^\infty drr^2 \left\{g_{(2s^{1/2})}(r)f_{(2p^{1/2})}(r) + f_{(2s^{1/2})}(r)g_{(2p^{1/2})}(r)\right\}$$
$$= \sqrt{\frac{k_C^2 - W_C^2}{4W_C^2 - k_C^2}} \left(\frac{W_C}{k_C} - \frac{k_C}{2W_C}(s+1)\right), \tag{196}$$

where formula $\int_0^\infty drr^{\nu-1}\exp(-\mu r) = \Gamma(\nu)/\mu^\nu$ were used. Consequently, substituting Eq.(196) into Eq.(195), we obtain

$$\langle H_1' \rangle^{1/2,\,-1/2}_{2s^{1/2},2p^{1/2}} \equiv -i\Theta_1$$
$$= -i\frac{Q^1Q^0}{4R^2}\frac{W}{3}\sqrt{\frac{k_C^2 - W_C^2}{4W_C^2 - k_C^2}} \left(\frac{W_C}{k_C} - \frac{k_C}{2W_C}(s+1)\right), \tag{197}$$

which is just desired result of Eq.(118), and all above calculations have been checked by the *Mathematica*.

2. By means of similar calculations we get also that

$$\langle H_1' \rangle^{-1/2,\,1/2}_{2s^{1/2},2p^{1/2}} = -i\Theta_1. \tag{198}$$

Since $H_1' = H_1'^\dagger$, we have

$$\langle H_1' \rangle^{-1/2,\,1/2}_{2p^{1/2},2s^{1/2}} = (\langle H_1' \rangle^{1/2,\,-1/2}_{2s^{1/2},2p^{1/2}})^* = i\Theta_1, \tag{199}$$
$$\langle H_1' \rangle^{1/2,\,-1/2}_{2p^{1/2},2s^{1/2}} = (\langle H_1' \rangle^{-1/2,\,1/2}_{2s^{1/2},2p^{1/2}})^* = i\Theta_1. \tag{200}$$

Furthermore, to all other elements of H_1'-matrix, since $\int_0^{2\pi} d\phi\ \exp(\pm in\phi) = 0$ and $\int_0^\pi d\theta \sin\theta\cos^{2n+1}\theta = 0$ and etc, the explicit calculations show that all those H_1'-matrix elements vanish. Consequently, all elements of H_1' are calculated, and Eq.(118) is proved.

(II) H_2'-matrix elements:

H_2' has been given in Eqs. (122) (123) in the text, which is as follows

$$H_2' = \frac{Q^1Q^0}{4R^2}\left(i\hbar c\overrightarrow{\frac{\partial}{\partial x^1}} - i\hbar c\overleftarrow{\frac{\partial}{\partial x^1}}\right), \tag{201}$$
$$\text{where}\quad \frac{\partial}{\partial x^1} \equiv \partial_1 = \sin\theta\cos\phi\frac{\partial}{\partial r} + \cos\theta\cos\phi\frac{1}{r}\frac{\partial}{\partial\theta} - \frac{\sin\phi}{r\sin\theta}\frac{\partial}{\partial\phi}. \tag{202}$$

We derive Eq.(124) in text now.

1. $\langle H_2'\rangle^{1/2,\,-1/2}_{2s^{1/2},2p^{1/2}}$:

$$\langle H_2'\rangle^{1/2,\,-1/2}_{2s^{1/2},2p^{1/2}} = \int drr^2 \int d\Omega\ \psi^{1/2\dagger}_{(2s)j=1/2}(\mathbf{r})\ H_2'\ \psi^{-1/2}_{(2p)j=1/2}(\mathbf{r})$$
$$= -\frac{Q^1Q^0}{4R^2}i\hbar c\int drr^2\int d\Omega\Big(g_{(2s^{1/2})}(r)\chi^{1/2\dagger}_{\kappa}(\hat{\mathbf{r}})_{(2s^{1/2})}, -if_{(2s^{1/2})}(r)\chi^{1/2\dagger}_{-\kappa}(\hat{\mathbf{r}})_{(2s^{1/2})}\Big)\Big(\overrightarrow{\partial}_1 - \overleftarrow{\partial}_1\Big)$$
$$\times\begin{pmatrix} g_{(2p^{1/2})}(r)\chi^{-1/2}_{\kappa}(\hat{\mathbf{r}})_{(2p^{1/2})} \\ if_{(2p^{1/2})}(r)\chi^{-1/2}_{-\kappa}(\hat{\mathbf{r}})_{(2p^{1/2})}\end{pmatrix}$$
$$= -i\frac{Q^1Q^0}{4R^2}\hbar c\int drr^2\int d\Omega\,\Big\{g_{(2s^{1/2})}(r)[\partial_1 g_{(2p^{1/2})}(r)]\chi^{1/2\dagger}_{\kappa}(\hat{\mathbf{r}})_{(2s^{1/2})}\chi^{-1/2}_{-\kappa}(\hat{\mathbf{r}})_{(2p^{1/2})}$$
$$+g_{(2s^{1/2})}(r)g_{(2p^{1/2})})(r)\chi^{1/2\dagger}_{\kappa}(\hat{\mathbf{r}})_{(2s^{1/2})}[\partial_1\chi^{-1/2}_{-\kappa}(\hat{\mathbf{r}})_{(2p^{1/2})}] + f_{(2s^{1/2})}(r)[\partial_1 f_{(2p^{1/2})}(r)]\chi^{1/2\dagger}_{\kappa}(\hat{\mathbf{r}})_{(2s^{1/2})}\chi^{-1/2}_{-\kappa}(\hat{\mathbf{r}})_{(2p^{1/2})}$$
$$+f_{(2s^{1/2})}(r)f_{(2p^{1/2})})(r)\chi^{1/2\dagger}_{\kappa}(\hat{\mathbf{r}})_{(2s^{1/2})}[\partial_1\chi^{-1/2}_{-\kappa}(\hat{\mathbf{r}})_{(2p^{1/2})}] - [\partial_1 g_{(2s^{1/2})}(r)]g_{(2p^{1/2})})(r)\chi^{1/2\dagger}_{\kappa}(\hat{\mathbf{r}})_{(2s^{1/2})}\chi^{-1/2}_{-\kappa}(\hat{\mathbf{r}})_{(2p^{1/2})}$$
$$-g_{(2s^{1/2})}(r)g_{(2p^{1/2})})(r)\chi^{1/2\dagger}_{\kappa}(\hat{\mathbf{r}})_{(2s^{1/2})}[\partial_1\chi^{-1/2}_{-\kappa}(\hat{\mathbf{r}})_{(2p^{1/2})}] - [\partial_1 f_{(2s^{1/2})}(r)]f_{(2p^{1/2})}(r)\chi^{1/2\dagger}_{\kappa}(\hat{\mathbf{r}})_{(2s^{1/2})}\chi^{-1/2}_{-\kappa}(\hat{\mathbf{r}})_{(2p^{1/2})}$$
$$-f_{(2s^{1/2})}(r)f_{(2p^{1/2})}(r)\chi^{1/2\dagger}_{\kappa}(\hat{\mathbf{r}})_{(2s^{1/2})}[\partial_1\chi^{-1/2}_{-\kappa}(\hat{\mathbf{r}})_{(2p^{1/2})}]\Big\}$$
$$= -i\frac{Q^1Q^0}{4R^2}\hbar c\left\{\frac{1}{3}\int_0^\infty drr^2\left[\frac{\partial g_{(2s^{1/2})}(r)}{\partial r}g_{(2p^{1/2})}(r) - \frac{\partial g_{(2p^{1/2})}(r)}{\partial r}g_{(2s^{1/2})}(r) + \frac{\partial f_{(2s^{1/2})}(r)}{\partial r}f_{(2p^{1/2})}(r)\right.\right.$$
$$\left.\left.-\frac{\partial f_{(2p^{1/2})}(r)}{\partial r}f_{(2s^{1/2})}(r)\right] - \frac{2}{3}\int_0^\infty drr\left[g_{(2s^{1/2})}(r)g_{(2p^{1/2})}(r) - f_{(2s^{1/2})}(r)f_{(2p^{1/2})}(r)\right]\right\}, \tag{203}$$

where the integrals to $d\Omega$ have been accomplished in terms of the explicit expressions of $\chi^{1/2}_{\pm\kappa}(\hat{\mathbf{r}})_{(2s^{1/2})}$, $\chi^{-1/2}_{\pm\kappa}(\hat{\mathbf{r}})_{(2p^{1/2})}$ in Eqs.(46) (47) (51) (52) and Eq.(202). Substituting expressions (44) (45) (49) (50) into Eq. (203), and finishing the integrals, we get

$$\langle H_2'\rangle^{1/2,\,-1/2}_{2s^{1/2},2p^{1/2}} \equiv -i\Theta_2$$
$$= i\frac{Q^1Q^0}{2R^2}\frac{\hbar c\lambda}{6\sqrt{4W_C^2-k_C^2}}\left(\frac{k_C^2}{W_C} - 2(\frac{1}{s}+1)k_C - W_C + \frac{2}{k_C s}W_C^2\right), \tag{204}$$

which is just Eq.(124), and all above result have been checked by the *Mathematica* calculations.

2. By means of similar calculations we get also that

$$\langle H_2'\rangle^{-1/2,\,1/2}_{2s^{1/2},2p^{1/2}} = -i\Theta_2. \tag{205}$$

Since $H_2' = H_2'^\dagger$, we have

$$\langle H_2'\rangle^{-1/2,\,1/2}_{2p^{1/2},2s^{1/2}} = (\langle H_2'\rangle^{1/2,\,-1/2}_{2s^{1/2},2p^{1/2}})^* = i\Theta_2, \tag{206}$$
$$\langle H_2'\rangle^{1/2,\,-1/2}_{2p^{1/2},2s^{1/2}} = (\langle H_2'\rangle^{-1/2,\,1/2}_{2s^{1/2},2p^{1/2}})^* = i\Theta_2. \tag{207}$$

Furthermore, to all other elements of H_2'-matrix, since $\int_0^{2\pi} d\phi\ \exp(\pm in\phi) = 0$ and $\int_0^\pi d\theta \sin\theta\cos^{2n+1}\theta = 0$ and etc, the explicit calculations show that all those H_2'-matrix elements vanish. Consequently, all elements of H_2' are calculated, and Eq.(124) is proved.

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[26] Footnote: This result is different from the author's former work arXiv: 1004.3023v4 [physics.gen-ph]. There is a mistake in the calculations of arXiv: 1004.3023v4 [physics.gen-ph]: the equations (53) (54) of arXiv: 1004.3023v4 should be corrected into Eqs.(16) (17) of this pesent paper.

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